

### 2nd International E-Conference on Mathematical and Statistical Sciences: A Selçuk Meeting

**Book of Abstracts** 

https://icomss23.selcuk.edu.tr

# 2nd INTERNATIONAL E-CONFERENCE ON MATHEMATICAL AND STATISTICAL SCIENCES: A SELÇUK MEETING (ICOMSS'23)

June 5 – June 7, 2023

Selçuk University, Konya https://icomss23.selcuk.edu.tr

## DEDICATED TO

# VICTIMS OF TÜRKİYE EARTHQUAKES

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### Proceedings

#### **Proceeding Book:**

• 2023 Proceedings of International E-Conference on Mathematical and Statistical Sciences: A Selcuk Meeting (ISBN: 978-625-00-8778-7)

#### **Special Issues:**

- Demonstratio Mathematica
- Journal of Mathematical Analysis
- Advanced Studies: Euro-Tbilisi Mathematical Journal
- Symmetry
- Applied General Topology

### Message from Chair

It's my pleasure to chair The International Conference on "2nd International E-Conference on Mathematical and Statistical Sciences: A Selçuk Meeting". Our 2023 conference, which is organized by the Faculty of Science of Selçuk University and supported by Scientific Research Projects Coordinatorship of Selçuk University. ICOMSS conference series was initiated in 2022 and first ICOMSS was held in 20-22 October 2022 online, see the webpage: https://icomss22.selcuk.edu.tr/.

Two earthquakes measuring 7.7 took place on 6 February 2023 at 04:17 in Pazarcık district of Kahramanmaraş and 7.6 took place at 13:24 in Elbistan district of Kahramanmaraş. It was felt intensely in the surrounding provinces, especially in Kahramanmaraş, Hatay, Osmaniye, Adıyaman, Gaziantep, Şanlıurfa, Diyarbakır, Malatya, Kilis and Adana. Unfournately, around 46000 people lost their life and 13,5 million people directly effected by the earthquakes. In order to meet the urgent needs of our citizens in the disaster many organizations, public institutions and international communities have supported the peoples. In this frame, we decided to organize second ICOMSS to be able to support the students and peoples effected by earthquakes. Therefore, the registartion fees of ICOMSS'23 donated to them. There is no registration fee for students and academicians directly effected by earthquakes.

By organizing this e-conference series, our main was the to promote, encourage, and provide a forum for the academic exchange of ideas and recent research works. The conference present new results and future challenges, in a series of virtual keynote lectures and virtual contributed short talks. In our conferences, we provide a forum for mathematicians and statistician to communicate recent research results in the areas of Algebra, Analysis, Applied Mathematics, Geometry, Topology, Applied Statistics, Statistical Theory. The conference were only online. The all presentation language was English, and submissions were be peerreviewed by at least two referees. I hope to organize this Conference in our university Konya, Turkiye and see you all.

I am thankful to the management of Selçuk University for providing the necessary support to organize this event. I am also thankful to all scientific committee members, organizing committee members, the session chairs, and the numerous volunteers, without their generous contributions this conference would not have set this number of presentations and participants.

Thanks.

#### Prof. Dr. Semahat Küçükkolbaşı

Dean of Faculty of Science Selçuk University Chair of ICOMSS'23

# PROGRAMME

			ICOMSS'23 PROGRAMME																	
				5 JUNE 2023																
Time (GMT +03:00)	ID			MATHEMATICS																
	DNI			PROF. DR. METÍN AKSOY (RECTOR OF SELÇUK UNIVERSITY, HONORARY CHAIR)																
09.30-10.00	OPEN			PROF. DR. SEMAHAT KÜÇÜKKOLBAŞI (DEAN OF FACULTY OF SCIENCE, CHAIR)																
10.00-10.30	1	. Acar	Invited Speaker: Andrei Vesnin																	
10.30-11.00	2	Tuncei							Invite	ed Sp	beaker: Vladimii	r Vaske	vich							<u>Link</u>
									B	REAK	(10 MIN.)									
			\$1			S2 (Room 1)	-	S2 (Room 2)	Zoom Link		S3 (Room 1)	Zoom Link		S3 (Room 2)	Zoom Link	S	54		S5	Zoom Link
11.10-11.25	3					C. Ünal		T. Kesaho	rm	öl	G. Seyhan Öz	tepe		G. Topcu	l				D. Kesik	
11.25-11.40	4				ıslı	M. Selah	rac	N. Manav N	lutlu	Akg			ntar	F. A. Çuh	a			gin	N. Manav Ta	atar
11.40-11.55	5				ı Ka	E. Demir	izsa	D. Tülü		ğru	J. Rychta	r	Altır	F. Başçift	çi			Ği	Z. Güzel Erg	gül
11.55-12.10	6				arur	D. Özer	at ć	S. Bilgiç		å	J. A. Nescolarde	e Selva	ya /	M. Spasov	/a			cber	M. Ateş	
12.10-12.25	7				Ï	G. Başcanbaz Tunca	Ē	N. Şahin Ba	/ram	ibel	D. Öztürl	(	Der	T. Ceylar	1			Ť	A. Büyükka	ya
12.25-12.40	8	_				F. Dirik		N. S. Ura	1	S									L. Dönme:	z
12.40-12.55	9					H. Karslı		F. Özsara	iç	er	S. Hristov	а								
12.55-13.10	10					M. A. Okur	öz	H. Gürkö	k	Pek	E. Yıldırım Ka	vgacı								
13.10-13.25	11				jnal	S. Yıldız	lag	N. Manjavi	dze	slan	B. Oğul									
13.25-13.40	12				an C	S. Çınar	d u	A. Yalçın	1)	par	J. Alzabu	t								
13.40-13.55	13				Cih	İ. Kura	, in the second sec	L. Aiemsomb	oom	n Al	R. Chemla	al								
13.55-14.10	14					E. Gül	0	S. Ekici		nple	A. Yetim									
14.10-14.25	15					T. Bostancı		S. Ozcan	1)	Ĩ	R. Terziev	а								
									В	REAK	(15 MIN.)									
14.40-15.10	16	Vltintan							Inv	vited	Speaker: Hayda	ar Bulga	ık							<u>Zoom</u>
15.10-15.40	17	Derya /							Inv	vited	Speaker: D.G. F	Prakash	а							<u>Link</u>

ID	Titles of the talks								
1	Andrei Vesnin	Presentations of Flat Virtual Groups by Automorphisms of Free Groups							
2	Vladimir Vaskevich	Spherical Polyharmonic Equation and Related Topics							
16	Haydar Bulgak	Graphics Constructor 4.0							
17	D.G. Prakasha	Static Perfect Fluid Space-time and Paracontact Metric Geometry							

		S2 (Room 1)	ID	ID S2 (Room 2)			
3	C. Ünal	On Existence and Multiplicity of Solutions for a Biharmonic Problem With Weights via Ricceri's Theorem	3	T. Kesahorm	On Novel Semigroup of Enriched Chatterjea Type Mappings in Banach Spaces		
4	M. Selah	On the Existence of Solutions for Multi-valued Operator Inclusions	4	N. Manav Mutlu	A New Generalization of Stancu-schurer Operators		
5	E. Demir	Unbounded Convergence in Cp(X, [0, 1])	5	D. Tülü	On Unbounded Order Continuous Urysohn Operators		
6	D. Özer	Convergence of a Family of Bivariate Sampling-Durrmeyer Operators in Weighted Spaces of Functions	6	S. Bilgiç	Second Type Parametric Unified Apostol-Bernoulli Polynomials		
7	G. Başcanbaz Tunca	On Kantorovich Variant of Brass-Stancu Operators	7	N. Şahin Bayram	On Power Series Strong and Statistical Convergence for Double Sequences via Orlicz Function		
8	F. Dirik	Statistical Korovkin Type Approximation Theorems for Double Sequences of Monotone and Sublinear Operators for Bögel	8	N. S. Ural	Fractional Trigonometric Korovkin-type Results by Statistical Convergence Based on a Power Series Method		
9	H. Karslı	Approximation Process of a New Operators of Hypergeometric Type	9	F. Özsaraç	On the Mellin-Gauss-Weierstrass Operators Preserving Logarithmic Functions		
10	M. A. Okur	Some Tauberian Conditions for the Logarithmic Integrability	10	H. Gürkök	On Unbounded Order Continuous Operators 2		
11	S. Yıldız	Approximation Theorems of Korovkin Type for Sequences of Monotone and Sublinear Operators via Power Series Method	11	N. Manjavidze	Generalized Meromorphic Functions		
12	S. Çınar	Approximation via Power Series Statistical Convergence of Matrix- valued Fuctions	12	A. Yalçın (1)	Some Novel Estimations of Hadamard Type Inequalities for Different Kinds of Convex Functions via Tempered Fractional Integral Operator		
13	İ. Kura	Classification of Color Spaces in Various Combinations Using Deep Learning Models	13	L. Aiemsomboom	Stability Results of Generalized Fuzzy Number-valued Function Equations via a Metric Defined on a Fuzzy Number Space		
14	E. Gül	On a Second Regularized Trace Formula	14	S. Ekici	Multivariate Trigonometric Korovkin Theorem in Fuzzy Setting		
15	T. Bostancı	A New Generalization of Kantorovich Operators Depending on a Non- negative Integer	15	S. Özcan (1)	Hermite-Hadamard Type Inequalities for Multiplicatively (s,P)- Functions		
		S3 (Room 1)			S3 (Room 2)		
3	G. Seyhan Öztepe	Some Bifurcations of Fractional Order Discrete Prey-Predator Model	3	G. Topcu	An Approach to Schur Stable Matrix Families		
4			4	F. A. Çuha	Effectiveness of the Kashuri Fundo Decomposition Method in Solving Fractional Differential Equations: a Case Study on the		
5	J. Rychtar	Modeling Guinea Worm Disease	5	F. Başçiftçi	Mathematical Analysis of Ionospheric Anomalies Before and After the Mw 7.8, Southeastern Turkey Earthquake		
6	J. A. Nescolarde Selva	Modeling Complex Social Systems: a Vision Using Networks	6	M. Spasova	Fuzzy Sumudu Transform to Solve Convolution Type Volterra Fuzzy Integro-Differential Equations		
7	D. Öztürk	Minimizing the Pressure on Filter Surfaces Using Optimal Control	7	T. Ceylan	Fuzzy Initial Value Problem With Dirac Delta Function		
8			8				
9	S. Hristova	Stability of Impulsive Fractional Differential Equations With Generalized Proportional Riemann-Liouville Fractional Derivatives			•		
10	E. Yıldırım Kavgacı	Eigenvalues of an Impulsive Schrödinger Equation					
11	B. Oğul	On the Recursive Sequence $x_{n+1} = [x_{n-3}] / [1+x_{n}x_{n-1}] / [1+x_{n}x_{n-1}] / [1+x_{n-3}x_{n-3}x_{n-4}] / [1+x_{n-5}x_{n-6}] / [1+x_{n-7}] / [1+x_{n-1}x_{n-1}] / [1+x_{n-1}x_{n-1}] / [1+x_{n-1}x_{n-1}x_{n-1}] / [1+x_{n-1}x_{n-1}x_{n-1}x_{n-1}x_{n-1}] / [1+x_{n-1}x_$					

12	J. Alzabut	New Approach for Fractional Differential Equations of Variable Order
13	R. Chemlal	Combining Chaotic Dynamical Systems Using Xor Operator
14	Aylin Yetim	An Optimal Control Strategy to Decrease the Harmful Effects of Online Game Addiction
15	R. Terzieva	Overview of Different Types of Stability of the Solutions of Differential Equations With Non-instantaneous Impulses
		S4
3	D. Kesik	Some Contraction Mappings With Binary Relation in Partial Modular b-metric Spaces
4	N. Manav Tatar	Deconstructing Fixed Points Theorems in Modular Metric Spaces
5	Z. Güzel Ergül	A Novel Type of Inverse Soft Covering Based Rough Set and Its Application to Decision Making
6	M. Ateş	An Approach Structure Related With Fell Topology
7	A. Büyükkaya	Some Fixed Point Results in R-modular b-metric-like Space
8	L. Dönmez	On Some Fixed Point Results Through Admissible Mappings in Quasi- partial b-metric Spaces

			ICOMSS'23 PROGRAMME	
			5 JUNE 2023	
Time (GMT +03:00)	ID		STATISTICS	
09 20 10 00	DNIN	Р	ROF. DR. METİN AKSOY (RECTOR OF SELÇUK UNIVERSITY, HONORARY CHAIR)	Zoom
09.30-10.00	OPEN	F	PROF. DR. SEMAHAT KÜÇÜKKOLBAŞI (DEAN OF FACULTY OF SCIENCE, CHAIR)	<u>Link</u>
17.00-18.00	1	an İyit	Invited Speaker: Narayanaswamy Balakrishnan	<u>Zoom</u>
18.00-19.00	2	Neslih	Invited Speaker: Barry Arnold	<u>Link</u>

ID		Titles of the talks
1	Narayanaswamy Balakrishnan	Family of Mean-Mixtures of Multivariate Normal Distributions
2	Barry Arnold	Lorenz Order With Common Finite Support

F				ICOMSS'23 PROGRAMME													
F	Time	ID								6 JUNE 202: MATHEMATI	<u>s</u> CS						
F	(GMT +03:00)		ē														
	09.30-10.00	1	A. Pek		Invited Speaker: Venkatesha Venkatesha											<u>Zoom</u>	
	10.00-10.30	2	Haldun					Ir	vite	d Speaker: Yuri	Luchko	)					<u>Link</u>
					-	1		В	REAK (	(10 MIN.)	-						
				<b>S1</b>	Zoom Link		S2	Zoom Link		S3	Zoom Link		S4	Zoom Link		S5	Zoom Link
	10.40-10.55	3		Z. Özkı	ırt		A. Yalçın (2	2)		E. Kırlı			D. Sağlar	n	rk		
	10.55-11.10	4	æ	H. Gülsün Akay	dur	M. Et		yar	D. Şehitoğul	ları	ä	H. Wang	5	Ďztü			
	11.10-11.25	5	Kay	G. Kahrım	G. Kahrıman (1)	K. H. Alan	n	o Ka	Z. Sakarter	be	pzde	H. Kadıoğ	lu	er ć	E. Girgin (	[1]	
	11.25-11.40	6	zgi	M. M. O	skay	E. Aydoğan E. Aydoğan E. Aydoğan E. Aydoğan E. Aydoğan E. Aydoğan	ra Č	İ. Temli		beyk	V. Öztürk (1)						
	11.40-11.55	7		P. Oma	an	ž	S. Ersan		Zey	M. Lafcı Büyükka	hraman	Zeh	A. Çalışka	in	ahp	N. Taş	
	11.55-12.10	8		B. Gülmez	Temür		Ö. Gürel Yılr	maz		Y. Çin			M. Öcal		Σ	E. Kaplan	
	12.10-12.25	9		H. İz Öng	ören	_	Ö. Dalmano	ğlu		A. Doğan Ça	lışır		H. Altın Erd	em		T. Simsekler D	Dizman
Γ	12.25-12.40	10	Ś	İ. A. Öğ	üt	ran	H. E. Altır	ı	gac	Ş. Kutlu Ser	rin	c	Ö. Aksu			M. Almaha	ariq
Γ	12.40-12.55	11	üne	H. Arslan		Bay	Z. Kalkan	l .	Kav			rsla	P. İnselö	Z	aş	T. Altunö	İz
	12.55-13.10	12	Ak	N. J. Farida F. Infusino A. Emin	hin	B. Güntür	k	nre	M. Odabaşı Kö	prülü	nila	Ş. Afşar		Γlar	H. Taşbozar	n (1)	
	13.10-13.25	13	hat		G. Mutlu	l	aEr	C. Vural		azın	G. Özkan Tük	el (1)	Ξ.	A. S. Özkapı	u (1)		
	13.25-13.40	14	z		G. S. Kele	ş	Mus	H. Armağa	n	Ŷ	H. Ceyhan	(1)		Ş. Onbaşıo	ğlu		
	13.40-13.55	15		K. Yılm	az	-	N. Çetin			B. Aydın			G. Koru Yüce	kaya		E. Girgin (	(2)
								В	REAK (	(15 MIN.)							
	14.10-14.40	16	Özlem Acar			Invited			Spea	aker: Wutiphol	Sintuna	avar	at				<u>Zoom</u> Link
						_		В	REAK (	(10 MIN.)							
	14.50-15.05	17		D. Defi	ta		F. Temizsi	u	alan	E. Ata (1)			A. Leven	t	аг	A. S. Özkapı	u (2)
	15.05-15.20	18		G. Kahrım	an (2)	_	S. Herden	n	akç	S. Doğru Ak	göl	Уa	A. Torur	1	Yal	V. Öztürk	(2)
	15.20-15.35	19	eş	E. İğd	e	Altın	E. Acar		(ayn	S. Sakarya	a	eka	T. Yağmı	ır	erit	T. Vergil	i
	15.35-15.50	20	gün	L. C. Wan	ditra	Jan	M. Bodur	r	Ť	Z. Bekiryaz	ICI	Υüc	M. Mak		"	A. Turab	)
	15.50-16.05	21	t Ak	I. Yalğı	n	n Er	İ. Gökcan	İ. Gökcan	Bi	B. Ateş		oru	E. Yanık		3	H. Taşbozar	n (2)
	16.05-16.20	22	liha	R. Karal	kuş	seyi	N. Yılmaz	2	bcu	Ö. Akçay Kara	akuş	γ Kc	G. Uzun		kap	F. Yalaz	
	16.20-16.35	23	Z	İ. Şentü	irk	Ë	G. Uysal	_	10	Ö. Arıbaş		Güla	H. Çetinka	ya	.öz	E. Korkma	az
	16.35-16.50	24		E. Önal	Kır		M. Kısako	bl	üneı	Z. Kayar					A.S	E. Erdoğa	in
	16.50-17.05	25							G	B. Kaymakça	lan						

ID	) Titles of the talks								
1	Venkatesha Venkatesha	Certain Type of Metrics Related to Ricci Solitons on Almost Kenmotsu Manifolds							
2	Yuri Luckho	General Fractional Integrals and Derivatives and Their Applications							

16	Wutiphol Sintunavarat	Several Novel Approaches Obtained From Fixed Point Technique to Solve Linear and Nonlinear Problems
ID		51
3	Z. Özkurt	Normal Automorphisms on Leibniz Algebras
4	H. Gülsün Atay	Groupoid Structure on Homotopies of Simplicial Maps
5	G. Kahrıman (1)	Bimultipliers of R-Algebroids
6	M. M. Oskay	On Ternary Cyclic Codes With Two Zeros
7	P. Oman	Minimal Generators of Syzygy Modules and Colon Ideals via Matrices
8	B. Gülmez Temür	Some Permutation and Complete Permutation Polynomials Over Finite Fields
9	H. İz Öngören	A Class of LCD Codes Based on Circulant Matrices
10	İ. A. Öğüt	Algebra of Fibred Relations
11	H. Arslan	A New Approach to Macmahon's Equidistribution Theorem
12	N. J. Farida	The Characteristic of Endomorphism Ring End\((\mathbb{Z}_{p^m} \times \mathbb{Z}_{p})\) and Its RSA Variant
13	F. Infusino	Finitary Simplicial Complexes and Various Models From Commutative Algebra
14	A. Emin	On Square Centered Square Numbers
15	K. Yılmaz	Barr Exactness of Quadratic Modules Over Nil(2)-modules
17	D. Defita	On The Existence of MDS Matrices Over \$\mathbb{F}_{p}+v\mathbb{F}_{p}\$
18	G. Kahrıman (2)	Homotopies of Crossed Modules of Bimultipliers of R- Algebroids
19	E. İğde	Braided crossed module of Lie algebras via tensor product of crossed complexes over Lie algebroids
20	L. C. Wanditra	Artificial Neural Network Using Quiver Representation by Finite Cyclic Group
21	I. Yalğın	Equivalence Between 2-crossed Modules of R-algebroids and Simiplicial R-algebroids With Moore Complex Lenght Two
22	R. Karakuş	On the Laplacian and Laplacian-like Energies of Some Classes of Bicyclic and Tricyclic Graphs
23	İ. Şentürk	A Graph Structure by Means of Compliment Annihilator in Many-valued Algebras
24	E. Önal Kır	On a Generalization of \$\oplus _{\delta _{ss}}\$-supplemented Modules
ID		S2

2	A Valcin (2)	New Estimations for Chebyshev Type Inequalities via
3	A. falçılı (2)	Generalized Proportional Fractional Integral Operators
4	M Ft	On Deferred Statistical Boundedness of Generalized Difference
-	111.20	Sequences
5	K H Alam	Solution of an Algebraic Linear System of Equations Using
,		Fixed Point Results in C –algebra Valued Extended Branciari
6	E. Aydoğan	Unbounded Order Convergence Properties of Operator Nets
7	S. Ersan	On Generalizations of Convergence in Neutrosophic Normed Spaces
8	Ö. Gürel Yılmaz	Some Properties of \$q\$-Durrmeyer Operators
9	Ö. Dalmanoğlu	On a Generalization of the Operators Involving Apostol- Genocchi Polynomials
10	H. E. Altın	On Multidimensional Urysohn Type Meyer-König and Zeller Operators
11	Z. Kalkan	A Fixed Point Result for T-mean Nonexpansive Mappings in b- metric-like Spaces With an Application
12	B. Güntürk	Hyperstonean Spaces
13	G. Mutlu	Odd-order Symmetric Operators on Compact Star Graphs
14	G. S. Keleş	Factorization of Strongly Order Bounded Operators and Their Demi Class
15	N. Çetin	Approximation by Complex Modified Stancu Operators
17	F. Temizsu	Deferred Statistical Convergence and Deferred Statistical Continuity in Locally Solid Riesz Spaces
18	S. Herdem	Ibragimov-Gadjiev Operators Preserving Exponential Functions
19	E. Acar	Note on Approximation of Truncated Baskakov Operators by Fuzzy Numbers
20	M. Bodur	On Jain Operators
21	İ. Gökcan	Suborbital Graphs Obtained by Different Matrix Multiplications and Fibonacci and Pell Numbers
22	N. Yılmaz	A New Class of Exact Penalty Function Method for Inequality Constrained Optimization Problems
23	G. Uysal	On Modified Mellin–Picard Operators
24	M. Kısakol	Bivariate Operators that Reproduce Exponential Functions
ID		53
3	E. Kırlı	A Novel Computational Scheme for Numerical Solution of the Regularized Long Wave Equation
		A Study on the Condition Number of the Vandermonde Matrix
4	D. Şehitoğulları	With the Mock-Chebyshev Nodes
		Stancu Type Dunkl Generalization of Szasz Kantrovich
5	Z. Sakartepe	Operators Including Two Variable Hermite Polynomials

6	D. Altıntan	Hybrid Modeling Approaches for Biochemical Reaction				
7	M. Lafcı Büyükkahraman	A Mathematical Modeling of Post-Myocardial Infarction				
8	Y. Çin	Two-Variable Higher-order Generalized Fubini Polynomials				
9	A. Doğan Çalışır	A Variational Method Based on the Discrete q-hermite II Polynomials to Obtain Spectrum of a q-deformed Schrödinger Equation				
10	Ş. Kutlu Serin	Lototsky Chlodowsky Operators				
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13	C. Vural	On SIR Models With Fractional Derivatives				
14	H. Armağan	Fuzzy Logic Based Optimization of Multivariate Nonlinear Regression Models and Applying				
15	B. Aydın	On Maximal Zagreb Index				
17	E. Ata (1)	Generalized Fourier Transform: Illustrative Examples and Applications to Differential Equations				
18	S. Doğru Akgöl	Existence of Solutions for Second Order Impulsive Boundary Value Problems on Time Scales				
19	S. Sakarya	An Optimal Control Strategy to Reduce the Gadget Addiction				
20	Z. Bekiryazıcı	Analysis of Covid-19 Dynamics in Turkey Using Sinusoidal Transmission Rates and Stochastic Differential Equations				
21	B. Ateş	Electromagnetic Plane Wave Scattering from Perfect Electric Conducting Triaxial Ellipsoid				
22	Ö. Akçay Karakuş	The Inverse Problem of Discontinuous Sturm-Liouville Operator by Spectral Data				
23	Ö. Arıbaş	An Inverse Problem For a Nonlinear Fractional Diffusion Equation				
24	Z. Kayar	Reversed Nabla Pachpatte Type Dynamic Inequlities				
25	B. Kaymakçalan	Reversed Diamond Alpha Pachpatte Type Dynamic Inequlities				
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4	H. Wang	Algebraic and Geometric Properties of a Family of Rational Curves				
5	H. Kadıoğlu	Jet Bundles as Whitney Sums				
6	İ. Temli	An Extended Calculus on the Hopf Superalgebra				

7	A. Çalışkan	Dual Magnetic Curves and Flux Ruled Surfaces
8	M. Öcal	Spinor Quasi Equations in Euclidean 3-Space
9	H. Altın Erdem	Some Results on Cartan Null and Pseudo Null Bertrand Curves in Minkowski 3-space
10	Ö. Aksu	Some Characterizations on Radical Lightlike Hypersurfaces of Almost Product-like Lorentzian Manifolds
11	P. İnselöz	On Para-sasaki-like Manifolds Equipped With Generalized
12	Ş. Afşar	Some Geometric Properties on Almost Bronze Riemannian Manifolds
13	G. Özkan Tükel (1)	A Variational Problem on the Dual Pseudo-sphere
14	H. Ceyhan (1)	Generalized Geometric Phase Applications in the Optic Fiber With the Kinematics of Framed Curve
15	G. Koru Yücekaya	On the Algebra of Commutatitive Quaternions and Hamilton Matrices
17	A. Levent	Trigonometric Cubic Bezier Transition Curves
18	A. Torun	On the Invariant Submanifolds of Almost Bronze Riemannian Manifolds
19	T. Yağmur	On Gaussian Leonardo Hybrid Polynomials
20	M. Mak	Generalized Framed Helices in Euclidean 3-Space
21	E. Yanık	Evolution of the Electric Field via Elliptical Frame: Application With Darboux Vector and Helical Trajectories
22	G. Uzun	Ruled Surfaces Created by Successor Frames on the T_1N_1 Smarandache Curve Obtained From the Tangent and Normal Vectors of the Successor Curve
23	H. Çetinkaya	On the Algebra of Elliptical Dual Quaternions and Hamilton Matrices
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5	E. Girgin (1)	On Quasi Modular b-metric Spaces
6	V. Öztürk (1)	Some Fixed Point Theorems in F-metric Spaces
7	N. Taş	Triple-Composed S-Metric Spaces
8	E. Kaplan	Fixed-Circle Results via Bilateral Multi-valued Contractions
9	T. Simsekler Dizman	Picture Fuzzy Soft Topological Spaces and an Application of Multicriteria Group Decision Making Problems

10	M. Almahariq	On Proximal Homotopy and Proximal Path Homotopy in Computational Proximity
11	T. Altunöz	Constructions of some minimal exotic 4-manifolds
12	H. Taşbozan (1)	Bipolar Soft Expert Sets in Decision Making Problem
13	A. S. Özkapu (1)	New Fixed Point Results in Weak Partial Metric Spaces
14	Ş. Onbaşıoğlu	$(\alpha, \beta)$ - Contractive Mapping in Intuitionistic Fuzzy Metric-Like Spaces and Fixed Points
15	E. Girgin (2)	Fixed points of simulative contraction on non-Archimedean quasi modular b-metric spaces
17	A. S. Özkapu (2)	Some Fixed Point Results on Ultrametric Spaces Endowed With Graph
18	V. Öztürk (2)	Common Fixed Point Theorems In b-Metric Spaces
19	T. Vergili	Proximity on Digital Topological Spaces
20	A. Turab	On a Unique Solution of the Generalized Functional Equation Arising in Mathematical Psychology and Theory of Learning
21	H. Taşbozan (2)	Bipolar Soft Expert Sets on Nearness Approximation Space
22	F. Yalaz	A New Decomposition of Continuity via the Localization
23	E. Korkmaz	The Images of Quasi-Sober Diframes Under Certain Morphisms
24	E. Erdoğan	The New Perspective to Fixed Point Theory on Weak Partial Metric Space

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			6 JUNE 2023	
Time (GMT +03:00)	ID		STATISTICS	
10.00-11.00	1	Coşkun Kuş	Invited Speaker: Hon Keung Tony Ng	<u>Zoom</u> Link

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1	Hon Keung Tony Ng	Recent Advances in Statistical Analysis of System-based Reliability Data

			ICOMSS'23 PROGRAMME													
			7 JUNE 2023													
Time (GMT +03:00)	ID		MATHEMATICS													
09.30-10.00	1	er Acar					Ir	nvite	d Speaker: Rau	l Curto						<u>Zoom</u>
10.00-10.30	2	Tunce				Invited Speaker: Feng Dai									<u>link</u>	
		1					BI	REAK (	(10 MIN.)	T	T		T	T		1
			<b>S1</b>	Zoom Link		S2	Zoom Link		S3	Zoom Link		S4	Zoom Link		S5	
10.40-10.55	3		C. Dede			H. Leven	t		İ. Gölgeleye	en	ya	Z. Özdemir	(1)			
10.55-11.10	4	ε	S. Aktaş		gay	F. Yılmaz	Z	g	M. Akbaş		eka	H. B. Çolako	ğlu			
11.10-11.25	5	l Ca	İ. Y. Kibar		Tur	O. Altınta	ış	Tol	F. Şahantür	'k	۲üc	M. Çarboğ	a			
11.25-11.40	6	ura	T. Taş Adıyama	an	etin	S. Özcan (2)		iner	R. Sharma	l	2	F. Doğru				
11.40-11.55	7	>	Y. Balseven		Ň	C. Topuz	2	Gü	B. Yağlıpına	B. Yağlıpınar		Ş. N. Bozdağ	(1)			
11.55-12.10	8	_	Ö. Duran			Ö. Yazıcı	1		S. Birel		G	G. Özkan Tüke	el (2)			
12.10-12.25	9		E. Özel			S. Öztürl	<	5	M. E. Kavga	СІ						
12.25-12.40	10		D. M. Yayın		( <b>)</b>	I. Açık Demirci	vgac	F. Gölgeleye	en		H. İ. Arıcı					
12.40-12.55	11	am	M. Aydın		araç	G. Bozma	а	Ka	E. Ata (2)		ven	F. Aydoğmı	ış			
12.55-13.10	12	al C	O. Nohut		özs	İ. Çam		L I	E. Eroğlu		ĞÜ	T. Tamirci				
13.10-13.25	13	Vun	Y. Nacaroğlu		Irat	E. Ağyuz	2	Yild	M. Uzun		lkay	İ. Güven				
13.25-13.40	14		İ. Demir		Ē			nel	E. Şen		.—	Ş. N. Bozdağ	(2)			
13.40-13.55	15		B. Özmen									O. Ateş				
							BI	REAK	(15 MIN.)							
14.10-14.40	16	Harun Karslı	Invited Speaker: George Anastassiou								<u>Zoom</u> link					
	BREAK (10 MIN.)															
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15.05-15.20	18	E	D. Köse	kur	S. Kazımoğ	ģlu										
15.20-15.35	19	l Ca	E. G. Çolak		s.	Y. Özkar	1									
15.35-15.50	20	'ura	E. Kürkçü Çaka	ar	er	A. Kızıltep	be									
15.50-16.05	21	>	H. Dumlu Pola	at	ö	N. D. Ara	ıl									
16.05-16.20	22		S. Şeyran	٥												

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16	George Anastassiou	q-deformed and Parametrized Half Hyperbolic Tangent Function Based Banach Space Valued Multivariate Multilayer Neural Network Approximations
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4	S. Aktaş	On Toeplitz Matrices With Generalized Oresme Numbers
5	İ. Y. Kibar	On Quaternions Whose Components Are Higher Order Generalized Fibonacci Numbers
6	K. Taş Adıyaman	lautomorphisms of Free Metabelian Leibniz Algebra of Rank n
7	Y. Balseven	Some New Properties of Line Graphs and Their Omega Invariants
8	Ö. Duran	On q-generalized Hyperharmonic Numbers With Two Parameters
9	E. Özel	A High-dimensional Categorical Perspective on 2-crossed Modules
10	D. M. Yayın	Narayana Numbers as Degress of Graphs
11	M. Aydın	On the Ramanujan-Nagell Equation x^2+3^a.11^b.17^c=2^r.y^n
12	O. Nohut	Graphs With Padovan Numbers as Degrees
13	Y. Nacaroğlu	On the Mostar Index of Zero-divisor Graphs Obtained From Commutative Rings
14	İ. Demir	On Nilpotent Leibniz Algebras With One Dimensional Leib Ideal

15	B. Özmen	Power Graphs of Some Graph Classes
17	S. Koca	On The Parity of Rencontres Numbers
18	D. Köse	Omega Invariant of Second and Third Power Graphs of Tadpole Graphs
19	E. G. Çolak	Gaussian Generalized Ernst Numbers
20	E. Kürkçü Çakar	Generalized Closure Operators on Bounded Lattices
21	H. Dumlu Polat	On Exchange Rings
22	S. Şeyran	On the Fuzzy Randic Energy and Fuzzy Randic Estrada Index of Fuzzy Graphs
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3	H. Levent	Some Important Operators on the Space of Discrete-time Interval Signals
4	F. Yılmaz	On Invariant Subspace of the Shift Operator on $D(\mu)$ Space
5	O. Altıntaş	Majorization by Starlike Functions With Respect to t-symmetric Points of Complex Order
6	S. Özcan	Simpson, Midpoint and Trapezoid Type Inequalities for Multiplicatively s-convex Functions
7	C. Topuz	On a Modification of Mellin Convolution Type Operators
8	Ö. Yazıcı	Squeezing Function and a Characterization of Polydisc
9	S. Öztürk	Solutions of Congruence Equations via an Imprimitive Action of Some Modular Subgroups
10	I. Açık Demirci	On Triple Sequences in Gradual 2-normed Linear Spaces
11	G. Bozma	On Bernstein Schurer Type Operators Preserving Exponential Function
12	İ. Çam	Nonlinear Fractional Boundary Value Problems on an Infinite Interval
13	E. Ağyuz	A Study on Fubini Type Polynomials

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17	O. Alagöz	Simultaneous Approximations of Sampling Series on Weighted Spaces of Function and Image Processing With Digital Topology Method
18	S. Kazımoğlu	Majorization Results for Subclasses of Meromorphic q-starlike Functions of Complex Order
19	Y. Özkan	Logarithmic Coefficients for Starlike Functions Associated With Generalized Telephone Numbers
20	A. Kızıltepe	Radii of the Lemniscate Starlikeness and Convexity of the Functions Including Derivatives of Bessel Functions
21	N. D. Aral	Difference Sequence Space of Fractional Order Generated by Bell Numbers
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3	İ. Gölgeleyen	An Inverse Problem for the Kinetic Equation in an Unbounded Domain
4	M. Akbaş	On a Modular Time Stepping Scheme of Velocity-Vorticity Method via Grad-Div Stabilization for the Navier-Stokes
5	F. Şahantürk	Spectral Analysis of Discontinuous Fractional Boundary Value Problem
6	R. Sharma	Mathematical Modeling of Detection, Spread and Control of Covid-19 Disease in Presence of Illegal Migrants
7	B. Yağlıpınar	The Solution of Interval Linear Equation System by Iterative Decreasing Dimension Method
8	S. Birel	Application of Exponential Smoothing and Auto-Regressive Integrated Moving Average (ARIMA) Mathematical Models to
9	M. E. Kavgacı	First Order Neutral Differential Equations With Piecewise Constant Mixed Arguments
10	F. Gölgeleyen	A Hybrid Numerical Algorithm for an Inverse Problem for the General Transport Equation
11	E. Ata (2)	Generalized Laplace Transform: Illustrative Examples and Applications to Differential Equations
12	E. Eroğlu	Bézier Cubics and Neural Network Agreement Along a Moderate Geomagnetic Storm

13	M. Uzun	On the Optical Soliton Solutions to Nonlinear Phenomena Arising in Communication Engineering
14	E. Şen	Non-local Sturm-Liouville Problems With Eigenvalue Parameter Contained in the Boundary Conditions
15		
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4	H. B. Çolakoğlu	Generalized Split Quaternions and Their Applications on Non- parabolic Conical Rotations
5	M. Çarboğa	Geometry of Dual Umbrella Matrices
6	F. Doğru	On Outer Billiards
7	Ş. N. Bozdağ (1)	Lightlike Hypersurfaces of Meta-Metallic Semi-Riemannian Manifolds
8	G. Özkan Tükel (2)	Curves Along Lorentzian Submersions
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10	H. İ. Arıcı	Some Results on Timelike Mannheim Curves in Minkowski 3- Space
11	F. Aydoğmuş	The Shortest Path is Not Always the Fastest
12	T. Tamirci	The Geometry of Almost Bronze Conjugate Connections
13	İ. Güven	Stationary Acceleration Curves via Sabban Frame
14	Ş. N. Bozdağ (2)	A Study on Hypersurfaces of Meta-Metallic Riemannian Manifolds
15	O. Ateş	Some Characterizations of Associated Curves in Minkowski 3- Space

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09.00-09.15	1		A.A.A. Gedik				
09.15-09.30	2	Jar	A. Yonar				
09.30-09.45	3	γo	İ. Şanlı				
09.45-10.00	4	nur	İ. Abusaif				
10.00-10.15	5	Ay	K. Karakaya				
10.15-10.30	6		E. Akgenç				
			BREAK (15 MIN.)				
10.45-11.00	7	ε	A. Bülbül				
11.00-11.15	8	iyra	T. Erbayram				
11.15-11.30	9	rba	O. Alkhuffash				
11.30-11.45	10	le	Y. C. Sevil				
11.45-12.00	11	E. Ertaş					
12.00-12.15	12	Τ	ー S. Sert				
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12.30-12.45	13	a	Ç. Çetinkaya				
12.45-13.00	14	alk	F. G. Akgül				
13.00-13.15	15	in K	A. Bektaş Kamışlık				
13.15-13.30	16	ppa	A. Bektaş Kamışlık				
13.30-13.45	17	ahre	M. Ağraz				
13.45-14.00	18	ű	M. Eriş Büyükkaya				
			BREAK (15 MIN.)				
14.15-14.30	19	ť	S. Кауа				
14.30-14.45	20	a Se	U. Mammadova				
14.45-15.00	21	eyra	A.Z. Dalar				

15.00-15.15	22	im	G. Eroğlu İnan				
15.15-15.30	23	Si	S. Gazioğlu				
	BREAK (30 MIN.)						
16.00-17.00	24	Coşkun Kuş	Invited Speaker: <u>Zoom</u> Haikady Nagaraja <u>Link</u>				
17.00	CLOSING CEREMONY						

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24	Haikady Nagaraja	Managing Concurrency in Cyclical Projects Under Stochastic Task Environments: Vaccine Development Projects During Pandemics							
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1	A.A.A. Gedik	A New Conditional Deductible in Insurance Policy							
2	A. Yonar	Designing a Pso-trained Ann Model for Estimation of Tourism Incomes							
3	İ. Şanlı	Statistical Inference for the Geometric Process With Skew-normal Distribution							
4	I. Abusaif	Multiple Arbitrarily Inflated Uniform-geometric Distribution and Its Application							
5	K. Karakaya	A Novel Count Regression Analysis Under Asymmetric Distribution							
6	E. Akgenç	Roc Curve Analysis for the Measurements Distributed Power-Lindley Distribution							
7	A. Bülbül	Independence Test for Family of Archimedean Copulas							
8	T. Erbayram	Estimation of the System Reliability for Poisson Ailamujia Distribution Based on Different Sampling Plans							

9	O. Alkhuffash	Estimation of Lifetime Performance Index for Power-Lindley
		Distribution Under Progressive First-failure Censoring
10	V C Sevil	Maximum Likelihood Estimates of Dependence Parameter in Fgm Type
10	1. C. SCVII	Bivariate Gamma Distribution From Ranked Set Sampling
		Investigation of the Effects of Gender and Coronary Artery Disease
11	E. Ertaş	(CAD) on Deaths in Patients Diagnosed With Covid-19 by Categorical
		Data Analysis Approach
12	S Sort	Point Estimation for the Inverse Rayleigh Distribution Under Type-II
12	5.5611	Left and Right Censoring
12	C. Cotinkovo	Inference for Overlapping Coefficient of Two Bathtub-shaped Lifetime
15	Ç. ÇELINKAYA	Distributions Under k-record Values
		Different Estimation Methods for Constant Stress Accelerated Life Test
14	F. G. Akgul	Under Exponentiated Pareto Distribution
		New Estimator for the Moments of a Stochastic Control Model Based
15	A. Bektaş Kamişlik	on Threshold Exceedances
10		Moment Based Approximation for a Semi-Markovian Inventory Model
16	A. Bektaş Kamışlık	With Asymmetric Triangular Distributed Interference of Chance
		Exploring Semi-supervised Machine Learning for Improved Diagnosis of
17	IVI. Agraz	Thyroid Disorders
18	M. Eriş Buyukkaya	A Note on Linear Mixed Models With Stochastic Restriction
	с. <i>к</i>	
19	S. Kaya	Statistical Inference for a New Alpha Power Topp Leone Distribution
		Exploring an Alternative Residual-based Control Chart for Monitoring
20	U. Mammadova	Overdispersed Count Profile
24		Ensemble Based Type-1 Fuzzy Functions Approach for Time Series
21	A.Z. Dalar	Forecasting
		Statistical Inference for Doubly Geometric Process With the Gamma
22	G. Eroglu Inan	Distributioness
		Sensitivity Analysis of a Deterministic Time-invariant Compartmental
23	S. Gazioğlu	Model

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# INVITED TALKS

### Q-deformed and Parametrized Half Hyperbolic Tangent Function Based Banach Space Valued Multivariate Multilayer Neural Network Approximations

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### **Abstract:**

Here we study the multivariate quantitative approximation of Banach space valued continuous multivariate functions on a box or  $\mathbb{R}^N$ ,  $N \in \mathbb{N}$ , by the multivariate normalized, quasi-interpolation, Kantorovich type and quadrature type neural network operators. We investigate also the case of approximation by iterated multilayer neural network operators of the last four types. These approximations are achieved by establishing multidimensional Jackson type inequalities involving the multivariate modulus of continuity of the engaged function or its partial derivatives. Our multivariate operators are defined by using a multidimensional density function induced by a q-deformed and parametrized half hyperbolic tangent function, which is a sigmoid function. The approximations are pointwise and uniform. The related feed-forward neural network are with one or multi hidden layers.

### Lorenz Order with Common Finite Support

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key-words: Finite support, Lorenz order.

### Abstract:

A special case of the Lorenz order of some interest is that in which the distributions being ranked with respect to inequality share a common finite support set. A characterization of this partial order is provided, together with the identification of operations that we call Robin Hood exchanges which play a parallel role in the finite support setting to that played by Robin Hood (or progressive) transfers in the general Lorenz ordering case. As an example of a setting in which variables with common finite support are compared with regard to inequality, two classical data sets involving social mobility are considered.

### **Family of Mean-Mixtures of Multivariate Normal Distributions**

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key-words: EM algorithm, likelihood estimation, multivariate normal distribution.

### Abstract:

In this talk, I will describe the family of mean-mixtures of multivariate normal distributions and establish many of its properties, stochastic representations, moments, distributional shape characteristics, etc. I will also describe a general EM-type algorithm for the likelihood estimation of model parameters. Finally, I will present some empirical results and a real data analysis, through a dataset that has been analyzed well in the literature.

### **Graphics Constructor 4.0**<sup>1</sup>

Haydar Bulgak<sup>1</sup>, Ali Osman Çıbıkdiken<sup>2</sup> and Dilaver Eminov<sup>3</sup>

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Key-words: computer dialogue application, spline functions, smart school.

### Abstract:

As a rule, the development of the new software for educational processes usually supports already published, independent textbooks. It is probably interesting to develop a software application in an interdependent relation to a textbook. Such an attempt was reflected in [1, 2, 3]. The focus in those publications was made to the floating-point numbers and the pixel structure of a computer screen. A table function and the first to third degree spline functions were chosen to represent the points by their given coordinates, straight lines, parabolas and curves of a third order to describe the main stages of constructing the sketches of the graphs of the real functions.

In contrast, an explanation of a graphical representation of a function using the terms like "competent chart" or "chart sketch" is given in the classical textbook [5] on page 35. These explanations follow with the tasks to build graphs and that immediately puzzles the student, as it is impossible to embrace the immensity of the unembracable.

In the Graphics Constructor 4.0<sup>2</sup>, the construction of a sketch of a graph of a function f begins with the selection of a rectangle  $[X_{min}, X_{max}] \times [Y_{min}, Y_{max}]$ . Then, the application allows you to choose the colour and thickness of the points and curves.

After defining the knots  $X_{min} < x_0 < x_1 < x_2 < ... < x_n < X_{max}$  and filling the table  $(x_k, f(x_k)), k = 0, 1, ..., n$ , the application allows you to draw a linear spline function. This is a sketch of the graphs of the given function.

Based on that sketch, the application can draw the sketches of the functions  $|f|, f^2, \frac{1}{f}$  and the inverse function. It also allows shifting the drawn graphs along the axes Ox and Oy independently.

After all, a sketch of a graph of a function is an image built with a finite sequence of points. Theoretically, it is possible to place the most irregular curves inside the blind zones, so to protect the actuality of the resulted sketch, the concepts of monotonicity, discontinuity, derivative, maximum and minimum of the derivative on the interval of the function are required. Hence, the application motivates the student to study these concepts.

- [1] Aydın K., Bulgak A. and Bulgak H., (2003), Mathematical Analysis via computer, Selcuk University, Research Centre of Applied Mathematics.
- [2] Bulgak A. and Eminov D., (2003), Graphics Constructor 2.0, Selcuk J. Appl. Math., Vol. 4(1), pp. 42-57.
- [3] Bulgak H., Eminov D. and Uçar M., (2019), Graphics Constructor 3.0, Konuralp Journal of Mathematics 7(2), pp. 333-336.
- [4] Demidovich B. P., (1997), Collection of tasks and exercises in mathematical analysis, 13th ed., Publishing House of Moscow, 644 p. (Russian)
- [5] Çıbıkdiken A.O., Bulgak A. and Eminov D., (2022), Graphics Constructor 4.0, Information Education and Method Electronic Learning: Digital Tecnologies in Education, Krasnoyarsk.

<sup>&</sup>lt;sup>1</sup> It is a replica of the presentation [5].

<sup>&</sup>lt;sup>2</sup>Graphics Constructor 4.0, an extended and improved version of Graphics Constructor 2.0 and Graphics Constructor 3.0 applications which were developed in Selcuk University.

### **Polynomial Embeddings of Unilateral Weighted Shifts in 2-variable Weighted Shifts**

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key-words: unilateral weight shift, 2-variable weighted shift, polynomial embedding.

### Abstract:

Given a bounded sequence  $\omega$  of positive numbers, and its associated unilateral weighted shift  $W_{\omega}$  acting on the Hilbert space  $\ell^2(\mathbb{Z}_+)$ , we consider natural representations of  $W_{\omega}$  as a 2-variable weighted shift, acting on the Hilbert space  $\ell^2(\mathbb{Z}_+^2)$ . Alternatively, we seek to examine the various ways in which the sequence  $\omega$  can give rise to a 2-variable weight diagram, corresponding to a 2-variable weighted shift. Our best (and more general) embedding arises from looking at two polynomials p and q nonnegative on a closed interval  $I \subseteq \mathbb{R}_+$  and the double-indexed moment sequence  $\{\int p(r)^k q(r)^\ell d\sigma(r)\}_{k,\ell \in \mathbb{Z}_+}$ , where  $W_{\omega}$  is assumed to be subnormal with Berger measure  $\sigma$  such that supp  $\sigma \subseteq I$ ; we call such an embedding a (p,q)-embedding of  $W_{\omega}$ . We prove that every (p,q)-embedding of a subnormal weighted shift  $W_{\omega}$  is (jointly) subnormal, and we explicitly compute its Berger measure.

We apply this result to answer three outstanding questions:

(i) Can the Bergman shift  $A_2$  be embedded in a subnormal 2-variable spherically isometric weighted shift  $W_{(\alpha,\beta)}$ ? If so, what is the Berger measure of  $W_{(\alpha,\beta)}$ ?

(ii) Can a contractive subnormal unilateral weighted shift be always embedded in a spherically isometric 2-variable weighted shift?

(iii) Does there exist a (jointly) hyponormal 2-variable weighted shift  $\Theta(W_{\omega})$  (where  $\Theta(W_{\omega})$  denotes the classical embedding of a hyponormal unilateral weighted shift  $W_{\omega}$ ) such that some integer power of  $\Theta(W_{\omega})$  is not hyponormal?

As another application, we find an alternative way to compute the Berger measure of the Agler *j*-th shift  $A_j$  ( $j \ge 2$ ). Our research uses techniques from the theory of disintegration of measures, Riesz functionals, and the functional calculus for the columns of the moment matrix associated to a polynomial embedding.

The talk is based on recent joint work with Sang Hoon Lee and Jasang Yoon.

### **General Fractional Integrals and Derivatives and Their Applications**

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key-words: general fractional integrals, general fractional derivatives of arbitrary order, regularized general fractional derivatives of arbitrary order, sequential general fractional derivatives, convolution Taylor formula, convolution Taylor series

### Abstract:

In this talk, some important results regarding the general fractional integrals and derivatives and the regularized general fractional derivatives presented in [1]-[8] are discussed. We start with a short historical survey of the general Fractional Calculus operators with the Sonin kernels and continue with a presentation of the recent developments devoted to this topic. In particular, we discuss the first and the second fundamental theorems of Fractional Calculus for the general fractional derivatives of arbitrary order, for the regularized general fractional derivatives of arbitrary order, and for the sequential general fractional derivatives. As an application of these results, the generalized convolution Taylor formulas and the generalized convolution Taylor series with the coefficients and remainders in terms of the general fractional derivatives and the regularized general fractional derivatives are presented.

- [1] Yu. Luchko, (2021), General Fractional Integrals and Derivatives with the Sonine Kernels, Mathematics, vol. 9(6), 594
- [2] Yu. Luchko, (2021), General Fractional Integrals and Derivatives of Arbitrary Order, Symmetry, vol. 13(5), 755
- [3] Yu. Luchko, (2021), Operational Calculus for the general fractional derivatives with the Sonine kernels, *Fract. Calc. Appl. Anal., vol.* 24(2), 338-375
- [4] Yu. Luchko, (2021), Special Functions of Fractional Calculus in the Form of Convolution Series and Their Applications, Mathematics, vol. 9(17), 2132
- [5] Yu. Luchko, (2022), Convolution series and the generalized convolution Taylor formula, *Fract. Calc. Appl. Anal., vol. 25,* 207-228
- [6] Yu. Luchko, (2022), Fractional Differential Equations with the General Fractional Derivatives of Arbitrary Order in the Riemann-Liouville Sense, *Mathematics, vol. 10(6), 849*
- [7] Yu. Luchko, (2022), The 1st level general fractional derivatives and some of their properties, J Math Sci, vol. 266, 709-722
- [8] Yu. Luchko, (2023), On the 1st-Level General Fractional Derivatives of Arbitrary Order, Fractal Fract., vol. 7(2), 183

### Managing Concurrency in Cyclical Projects Under Stochastic Task Environments: Vaccine Development Projects During Pandemics

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### Abstract:

Aggressive overlapping of stochastic activities during phases of vaccine development has been critical to making effective vaccines for COVID-19 available to the public, at "pandemic" speed. In cyclical projects wherein activities can be overlapped, downstream tasks may need rework on account of having commenced prior to receiving requisite information that is only available upon completion of upstream task(s). We provide a framework to understand the interplay between stochastic overlap duration and rework due to overlap, and its impact on minimizing expected completion time for a cyclical project. We find that planning overlapping in scenarios that may be deemed ineffective with an assumption of deterministic tasks, can be beneficial when analyzed using stochastic task duration. We determine optimal planned start times for stochastic tasks as a function of a parameter that proxies for the extent of net gain/loss from overlap to minimize expected completion time for the project.

(Joint work with Nagesh N. Murthy, University of Oregon, Eugene, USA, and Hossein Rikhtehgar Berenji, Pacific University, Oregon, USA)

### Recent Advances in Statistical Analysis of System-based Reliability Data

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key-words: Lifetime, reliability, optimal design.

### Abstract:

In system reliability engineering, systems are made up of different components, and these systems can be complex. For various purposes, engineers and researchers are often interested in the lifetime distribution of the system as well as the lifetime distribution of the components which make up the system. In many cases, the lifetimes of an n-component coherent system can be observed, but not the lifetimes of the components. In recent years, parametric and nonparametric inference for the lifetime distribution of components based on system lifetime lifetimes has been developed. In this talk, the recent development of statistical inference of the reliability characteristics of the components in the system based on the lifetimes of systems with the same structure will be discussed. First, we discuss the problem of testing the homogeneity of component lifetime distributions based on system lifetime data with known system signatures. Several nonparametric testing statistics based on the empirical likelihood method are proposed for testing the homogeneity of two or more component lifetime distributions. Both complete and Type-II censored system lifetime data will be considered. The performance of the proposed empirical likelihood ratio tests is compared with other parametric and nonparametric tests in the literature. Then, we study the optimal design of constant-stress life-testing experiments with ncomponent systems. Since experimental schemes with shorter experimental time and accurate statistical inference are desired, we consider putting the experimental units as n-component systems and propose schemes based on those n-component systems. Different experimental schemes based on n-component systems are considered, and the performances of these experimental schemes are compared via mathematical analysis and Monte Carlo simulation. The merits of the proposed experimental schemes based on n-component systems are discussed, and future research directions are provided.

- [1] Qu, J., Ng, H.K.T. and Moon, C. (2023), Empirical Likelihood Ratio Tests for Homogeneity of Component Lifetime Distributions Based on System Lifetime Data *submitted for publication*
- [2] Yu, Y. and Ng, H.K.T. (2023), Optimal Experimental Planning for Constant-Stress Accelerated Life-Testing Experiments based on Coherent Systems, to appear in Communications in Statistics Simulation and Computation.

### Static Perfect Fluid Space-time and Paracontact Metric Geometry

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key-words: K paracontact manifold,  $(k, \mu)$ -paracontact metric manifold, static space-time, Einstein manifold.

### Abstract:

The main purpose of thistalk is to discuss some characteristics of static perfect fluid space-time on paracontact metric manifolds. Here, we revealy the following:

- If a K-paracontact manifold  $M^{2n+1}$  is the spatial factor of a static perfect fluid space-time, then  $M^{2n+1}$  is of constant scalar curvature -2n(2n+1) and squared norm of the Ricci operator is given by  $4n^2(2n+1)$ .
- if a  $(k, \mu)$ -paracontact metric manifold  $M^{2n+1}$  with k > -1 is a spatial factor of static perfect space-time, then for  $n = 1, M^{2n+1}$  is flat, and for n>1,  $M^{2n+1}$  is locally isometric to the product of a flat (n + 1) -dimensional manifold and an n-dimensional manifold of constant negative curvature -4.
- if a paracontact metric 3-manifold  $M^3$  with  $Q\phi = \phi Q$  is a spatial factor of static perfect space time, then  $M^3$  is an Einstein manifold.
- Suitable example

### Several Novel Approaches Obtained From Fixed Point Technique to Solve Linear and Nonlinear Problems

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key-words: Brouwer fixed point theorem, Banach fixed point theorem, Fixed point, Tarski fixed point theorem.

### Abstract:

As precisely one century has transpired since the proof of the Brouwer fixed point theorem, which can be regarded as the precursor to other results, it is pertinent to reevaluate fixed point theorems in a broader context. It becomes evident that these theorems exhibit considerable diversity and pervasiveness throughout mathematics, encompassing algebra, analysis, geometry, topology, dynamics, number theory, group theory, and even set theory. Over time, numerous researchers in fixed point theory have endeavored to address fixed point problems by formulating various theorems tailored to specific scenarios. The fixed point theory is commonly categorized into three main domains: Topological fixed point theory, Metrical fixed point theory, and Discrete fixed point theory. Historically, the demarcation lines between these areas were established by discovering three significant theorems:

- The Brouwer fixed point theorem (see in [1]);
- The Banach fixed point theorem (see in [2]);
- The Tarski fixed point theorem (see in [3]).

This presentation give several techniques for solving linear and nonlinear problems, employing classical fixed point results and a range of other theorems.

### References

- [1] Brouwer, L.E., (1911), Beweis der invarianz der dimensionenzahl, Mathematische Annalen, vol.70, pp. 161-165.
- [2] Banach, S., (1922), Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales, *Fundamenta Mathematicae*, vol.3, pp. 133-181.
- [3] Tarski, A., (1955), A lattice-theoretical fixpoint theorem and its applications, *Pacific journal of Mathematics, vol.5, pp.285-309.*

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### **Spherical Polyharmonic Equation and Related Topics**

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key-words: Spherical partial differential equations, Laplace—Beltrami operator, polyharmonic equation, Sobolev-like spaces on the unite sphere.

#### Abstract:

Let S be the sphere of unit radius in  $\mathbb{R}^n$ ,  $n \ge 2$ . The projection of an arbitrary point x in  $\mathbb{R}^n$ ,  $x \ne 0$ , to S will be denoted by  $\theta$ ; i.e., we assume that  $\theta = x/\rho$ , where  $\rho = |x|$ . So  $\theta$  is a point in S. In what follows, the integrals over  $\theta$  are surface integrals over dS. Let us consider the differential equation of the form

$$-D)^m u(\theta) = p(\theta), \tag{1}$$

where D is the Laplace—Beltrami operator with respect to  $d\theta$  [1], m is a positive integer, and  $p(\theta)$  is a continuous function on S which obey the orthogonality condition  $\int p(\theta) d\theta = 0$ . The main results of the talk are about the solutions to (1). They are formulated in the following two theorems [2].

**Theorem 1.** Let m be an integer and m > (n-1)/2. Then for every functional  $l(\theta)$  in  $C^*(S)$  with (l, 1) = 0, the problem

$$(-D)^{m}u(\theta) = l(\theta), \quad \int u(\theta) \, d\theta = 0, \tag{2}$$

has a unique solution  $u(\theta)$  in the spherical Sobolev space  $H^m$ . For  $m \ge (3n-2)/4$  the solution to (2) belongs to the space  $C^{(2m-3n/2+1)}(S)$ .

The expansion of  $u(\theta)$  in the series has the form

$$u(\theta) = \sum_{k=1}^{\infty} \frac{1}{k^m (n+k-2)^m} \sum_{l=1}^{\sigma(k)} (l, Y_{k,l}) Y_{k,l}(\theta).$$

Here the set of functions  $\{Y_{k,l}(\theta) \mid l = 1, 2, \dots, \sigma(k)\}$  constitute an orthonormal basis for the space of spherical harmonics of order k:

$$\int Y_{k,l}(\theta) Y_{k,p}(\theta) \, d\theta = \delta_l^p.$$

**Theorem 2.** Let  $p(\theta)$  be a member of the spherical Sobolev space  $H^s$  for some s > (n-1)/2 and the equality  $\int p(\theta) d\theta = 0$  holds. Then there is a unique solution to the spherical polyharmonic equation

$$\left(-D\right)^{m} u(\theta) = p(\theta)$$

such that it is orthogonal to the identically-one function and belongs to the space  $H^q$  for q = s + 2m. The function  $u(\theta)$  can be written as follows

$$u(\theta) = \int G(\theta \cdot \theta') p(\theta') d\theta',$$

where the function  $G(\theta \cdot \theta')$  is the Green's function of  $(-D)^m$ 

The definition of  $G(\theta \cdot \theta')$  is as follows

$$G(\theta \cdot \theta') = \frac{1}{\sigma_{n-1}} \sum_{k=1}^{\infty} \frac{\sigma(k)}{k^m (n+k-2)^m} G_k^{(n)}(\theta \cdot \theta').$$

Here  $G_{k}^{(n)}$  is the normalized Hegenbauer polynomial.

For s > (n-1)/2 the series on the right-hand side of (3) converges absolutely and uniformly. For two points  $\theta$  and  $\theta^{(j)}$  in S the function  $G(\theta \cdot \theta^{(j)})$  is a solution to the equation

$$(-D)^{m}G(\theta \cdot \theta^{(j)}) = \delta(\theta - \theta^{(j)}) - \frac{1}{\sigma_{n-1}} \int \delta(\theta - \theta') d\theta'.$$

Spherical polyharmonic equation (1) with error functionals in the right hand side is very impotent in the theory of cubature formulas [3-4].

### References

- [1] Lizorkin P. I. and Nikolskii S. M., (1987): Approximation by spherical functions, Proc. Steklov Inst. Math., 173, 195-203.
- [2] Vaskevich V. L., (2017): Spherical polyharmonic equation, Functional Differential Equations, 24, No. 3–4, 157–166.
- [3] Vaskevich V. L., (2017): Spherical cubature formulas in Sobolev spaces, Siberian Mathematical Journal, 58, No. 3, 408-418.
- [4] Sobolev S. L. and Vaskevich V. L., (1997): The Theory of Cubature Formulas, Kluwer Academic Publishers, Dordrecht.

(3)

### Certain type of metrics related to Ricci solitons on almost Kenmotsu Manifolds

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Key-words: Almost Kenmotsu manifolds, Ricci solitons, \*-Ricci soliton, Gradient almost \*-Ricci soliton, Gradient  $\rho$ -Einstein soliton, Einstein manifolds.

### Abstract:

Geometric flows have become crucial part in Riemannian geometry and general relativity. During the last few years, geometric flows (as a class of important geometric PDEs) are enjoying rapid growth by providing new techniques of investigations in different directions of study in differential geometry, analysis and theoretical physics. Geometry of Ricci solitons is highly pursued subject not only because of its elegant geometry, but also because of its applications in different disciplines. The importance of Ricci solitons [1] comes from the fact that they correspond to self-similar solutions of the Ricci flow [2] and at the same time they are natural generalizations of Einstein metrics. The gradient  $\rho$ -Einstein solitons give rise to the self-similar solutions to a perturbed version of the Ricci flow, the so-called Ricci-Bourguignon flows [3], which unifies several particular cases well studied in the literature, such as gradient Ricci soliton and gradient Schouten solitons. Recently, a new research interest has appeared regarding the so called \*-Ricci soliton [4], which is an analogue of Ricci soliton in almost contact geometry.

The objective of present talk is to characterize almost \*-Ricci solitons and gradient  $\rho$ -Einstein solitons on certain class of almost Kenmotsu manifolds. First, we will recall the basic notion and some geometric properties of almost \*-Ricci solitons, gradient  $\rho$ -Einstein solitons and almost Kenmotsu manifolds. First, we consider Kenmotsu metric as \*-Ricci soliton and proved that soliton constant is zero. Also, we showed that if Kenmotsu metric is a gradient almost \*-Ricci soliton, then either it is Einstein or the potential vector field is collinear with the characteristic vector field. We constructed an example to show the existence of a \*-Ricci soliton and gradient almost \*-Ricci soliton on Kenmotsu manifolds. Next, we proved that if the metric of an almost Kenmotsu manifold with conformal Reeb foliation admits a gradient  $\rho$ -Einstein soliton, then either it is Einstein or the potential vector field. Moreover, we proved that if the metric of a  $(\kappa, \mu)'$ -almost Kenmotsu manifold with admits a gradient  $\rho$ -Einstein soliton, then the manifold is locally isometric to the Riemannian product  $\mathcal{H}^{n+1}(-4) \times \mathcal{R}^n$  and potential vector field is tangential to the Euclidean factor  $\mathcal{R}^n$ . Finally, we provided an example of an almost Kenmotsu 3-manifold which admits a gradient rho-Einstein soliton.

- [1] Chow B. and Knopf D., (2004), The Ricci flow: an introduction, mathematical surveys and monographs, 110. American Mathematical Society; 2004.
- [2] Hamilton R. S., (1988), The Ricci flow on surfaces, Mathematics and general relativity (Santa Cruz,CA, 1986). Contemp. Math., vol. 71, A.M.S., 1988, p. 237–62.
- [3] Bourguignon J.P., (1981), Ricci curvature and Einstein metrics. In: Global differential geometry and global analysis (Berlin, 1979), *Lecture Notes in Math.*, 838, 42–63
- [4] Kaimakamis G. and Panagiotidou K., (2014), \*-Ricci solitons of real hypersurfaces in non-flat complex space forms, *Journal* of Geometry and Physics, 86, 408–413.

### **Presentations of Flat Virtual Groups by Automorphisms of Free Groups**

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key-words: braid group, flat virtual braid group, automorphism of a free group.

### Abstract:

In 1925 Artin [1] defined braid groups  $B_n$  on n strands,  $n \ge 2$ , and found a representation of  $B_n$  by automorphisms of the free group  $\mathbb{F}_n$  of rank n. Braid groups are the very useful tool to study knots and links in the 3-sphere, since any knot or link can be defined as a closure of some geometric braid. In 1999 Kaufman [2] introduced the concepts of virtual knots and links, virtual braids, and virtual braid groups  $VB_n$  on n strands. Further generalizations of braid groups and their presentations by automorphisms of the free groups are presented and discussed in [3]. Two families of polynomial invariants for virtual knots, which arise from flat virtual knot invariants, were constructed in [5].

We will discuss flat virtual braid groups  $FVB_n$ ,  $n \ge 2$ , with generators  $\sigma_1, \ldots, \sigma_{n-1}, \rho_1, \ldots, \rho_{n-1}$  and the following defining relations:

$$\sigma_i^2 = 1, \qquad \rho_i^2 = 1, \qquad 1 \le i \le n-1, \\ \sigma_i \sigma_{i+1} \sigma_i = \sigma_i + \sigma_i \sigma_{i+1}, \quad \rho_i \rho_{i+1} \rho_i = \rho_{i+1} \rho_i \rho_{i+1}, \quad \rho_i \rho_{i+1} \sigma_i = \sigma_{i+1} \rho_i \rho_{i+1}, \qquad 1 \le i \le n-2, \\ \sigma_i \sigma_j = \sigma_j \sigma_j, \qquad \rho_i \rho_j = \rho_j \rho_j, \qquad \rho_i \sigma_j = \sigma_j \rho_j, \qquad |i-j| > 2.$$

Let  $\mathbb{F}_{2n}$  be the free group with free generators  $x_1, \ldots, x_n, y_1, \ldots, y_n$ .

We construct infinite families of homomorphisms  $\Theta_n : FVB_n \to \operatorname{Aut}(\mathbb{F}_{2n})$  which are local in the sense that  $\Theta_n(\sigma_i)$  acts nontrivially only on  $x_i$  and  $x_{i+1}$ , while  $\Theta_n(\rho_i)$  acts nontrivially only on  $x_i, x_{i+1}, y_i, y_{i+1}$ , where  $i = 1, \ldots, n-1$ . We give a condition when presentations  $\Theta_2$  are faithful and study  $\operatorname{Ker}(\Theta_n)$  for  $n \geq 3$ .

A.V.'s work was carried out in the framework of the Sobolev Institute of Mathematics project FWNF-2022-0004.

- [1] Artin E., (1925) Theorie der Zöpfe, Abh. Math. Semin Univ. Hamburg, vol. 4, pp. 47-72.
- [2] Kauffman L., (1999) Virtual knot theory, European J. Comb., vol. 20(7), pp. 663-690.
- [3] Bardakov V., Emelyanenkov I., Ivanov M., Kozlovskaya T., Nasybulov T., Vesnin A., (2022) Virtual and universal braid groups and their quotients, *Journal of Group Theory*, vol. 25(4), pp. 679-712.
- [4] Kaur K., Prabhakar M., Vesnin A., (2018) Two-variable polynomial invariants of virtual knots arising from flat virtual knot invariants, *Journal of Knot Theory and Its Ramifications, vol. 27(13), paper number 1842015.*

## ORAL PRESENTATIONS

### Multiple Arbitrarily Inflated Uniform-Geometric Distribution and Its Application

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key-words: Count data, inflated count regression model, Uniform-Geometric distribution, Zero-Inflated Poisson distribution

### Abstract:

In many real-world scenarios, researchers encounter count data that exhibit an unusually high number of some observations along with overdispersion. To effectively analyze such data, the zero-inflated regression model has emerged as a powerful statistical tool when we have overdispersed count data with excess zeros. Akdoğan et al. [1] proposed a new discrete distribution called the Uniform-Geometric distribution (UG). In this study, we introduce a new count regression model based on UG distribution, called the Multiple Arbitrarily Inflated Uniform-Geometric regression model (MAIUG). Some distributional properties are discussed for the proposed distribution. The maximum likelihood algorithm is derived to obtain the estimates of the unknown parameters. A Monte Carlo simulation study is carried out in order to evaluate the performance of the maximum likelihood estimators. Finally, a real data set is analyzed in order to determine the superiority of the proposed model among others.

- [1] Akdoğan, Y., Kuş, C., Asgharzadeh, A., Kınacı, İ., & Sharafi, F. (2016), Uniform-geometric distribution, *Journal of Statistical Computation and Simulation*, 86(9), 1754-1770
- [2] Gradshteyn, I. S., & Ryzhik, I. M. (2007), Table of Integrals, Series and Products, New York: Academic.

### **On Triple Sequences in Gradual 2-normed Linear Spaces**

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key-words: Triple sequence, gradual normed linear space, lacunary sequence, lacunary statistical convergence, 2-normed space.

### Abstract:

The concept of lacunary statistical convergence of triple sequences with respect to progressive 2-normed linear spaces is introduced in this research. We learn about its link to some inclusion and fundamental qualities. The notion of lacunary statistical Cauchy triple sequences is introduced in the conclusion, and it is demonstrated that it is equivalent to the idea of lacunary statistical convergence.

- [1] Kişi, Ö., Üstünsoy, N. and Gürdal, M., 2023, Some results on  $(\lambda, \mu)$ -statistical convergence of sequences in gradual *n*-normed linear spaces, *Ann. Fuzzy Math. Inform., in press.*
- [2] Ettefagh, M., Etemad, S. and Azari, F. Y., 2020, Some properties of sequences in gradual normed spaces, *Asian-Eur. J. Math.*, 13(4), 2050085. DOI: 10.1142/S1793557120500850.
- [3] Fridy, J. A., 1993, Lacunary statistical summability, J. Math. Anal. Appl., 173(2), 497-504.
- [4] Fortin, J., Dubois, D. and Fargier, H., 2008, Gradual numbers and their application to fuzzy interval analysis, *IEEE Trans. Fuzzy Syst.*, *16*(2), 388-402.
- [5] Gähler, S., 1965, Linear 2-normietre Raüme, Math. Nachr., 28, 1-43.
- [6] Mohiuddine, S. A. and Aiyub, M., 2012, Lacunary statistical convergence in random 2-normed spaces, *Appl. Math. Inf. Sci.*, 6(3), 581-585.
- [7] Şahiner, A., Gürdal, M. and Düden, F. K., 2007, Triple sequences and their statistical convergence, *Selçuk J. Appl. Math.*, 8(2), 49-55.
- [8] Savaş, E. and Gürdal, M., 2014, Generalized statistically convergent sequences of functions in fuzzy 2-normed spaces, *J. Intell. Fuzzy Systems*, 27(4), 2067-2075.
- [9] Gürdal, M., Şahiner, A. and Açık, I., 2009, Approximation theory in 2-Banach spaces, Nonlinear Anal., 71(5-6), 1654-1661.
- [10] Gürdal, M. and Açık, I., 2008, On I-Cauchy sequences in 2-normed spaces, Math. Inequal. Appl., 11(7), 349-354.

### Some Geometric Properties on Almost Bronze Riemannian Manifolds

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key-words: Almost bronze structure, adapted connection.

### Abstract:

Almost bronze structures are new structures defined on differentiable manifolds. The new almost bronze structures are polynomial structures with a structure polynomial of  $Q(\varphi) = \varphi^2 - m\varphi + I_d$  for  $m \in \mathbb{R} \setminus [-2, 2]$ . The metallic structures family does not include this newly described structure. In this study, we investigate the relationship between the almost bronze and almost product structures. We study adapted connections to almost bronze structures and almost bronze Riemannian structures.

- [1] Etayo F., Santamaría R., (2016), Distinguished connections on  $(J^2 = \pm 1)$ -metric manifolds, Arch. Math. Brno, 52(3), 159–203
- [2] Etayo F., Santamaría R., (2017), The well adapted connection of a  $(J^2 = \pm 1)$ -metric manifold, RACSAM, 111(2), 355–375
- [3] Etayo F., Santamaría R. and Upadhyay A., (2017), On the geometry of almost golden Riemannian manifolds, *Mediterranean Journal of Mathematics*, 14, 187
- [4] Özkan M., Doğan S., (2022), Almost bronze structures on differentiable manifolds, *Hindawi Journal of Mathematics*, 2022, Article ID 6940387, 11

### Exploring Semi-Supervised Machine Learning Algorithms for Improved Diagnosis of Thyroid Disorders

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key-words: Thyroid disease, machine learning, semi-supervised learning, FixMatch.

### Abstract:

Hyperthyroidism and hypothyroidism are health conditions characterized by an overactive or underactive thyroid gland, respectively. While individuals with normal hormone levels may not typically fall into these categories in their daily lives, they can still suffer from thyroid disease. In this study, we utilized an electronic health records dataset of thyroid patients, which included labeled data for hypothyroid and hyperthyroid individuals, as well as normal patients. To conduct our analysis, we created labeled (hyperthyroidism/ hypothyroidism) and unlabeled datasets by removing the labels of normal patients. As thyroid disorders can manifest even in individuals with normal results from diagnostic tests, it is important to consider additional factors. Subsequently, we used semi-supervised machine learning methods, such as FixMatch, and self-training methods to predict hypothyroidism and hyperthyroidism using both the labeled and unlabeled data. To assess the effectiveness of these approaches, we compared the results obtained from the semi-supervised learning models with those derived from supervised learning models, including Naive Bayes and Logistic Regression. The FixMatch algorithm demonstrated promising results, achieving high accuracy, F1-measure, and sensitivity in detecting thyroid disease. Our findings revealed a significant improvement in model performance when utilizing semi-supervised machine learning techniques, especially the FixMatch algorithm, compared to the supervised learning models.

### A Study on Fubini Type Polynomials

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key-words: Generating functions, positive linear operators, Fubini type polynomials.

### Abstract:

In this study, we introduced a generalization of positive linear operators involving the generating function of Fubini type polynomials. In addition, we explore some approximation properties of these operators. Finally, we produce some examples about the rate of convergence of these operators to certain functions by using Maple2023.

- [1] Icoz, G., Varma, S. and Sucu, S., 2016, Approximation by operators including generalized Appell polynomials, *Filomat*, 30 (2), 429-440.
- [2] Atakut, Ç. and Buyukyazici, İ., 2016, Approximation by Kantorovich-Szász type operators based on Brenke type polynomials *Numerical Functional Analysis and Optimization*, **37:12**, 1488-1502.
- [3] Menekşe Yılmaz, M., 2020, Approximation by Szász Type Operators Involving Apostol-Genocchi Polynomials, *CMES-Computer Modeling in Engineering and Sciences*, **130** (1), 2887-297.
- [4] Agyuz, E., 2022, A Study on Kantorovich Type Operator Involving Adjoint Euler Polynomials, Conference Proceedings of Science and Technology, 5 (1), 178-181.
- [5] Gupta, V. and Rassias, M. T., 2019, Moments of linear positive operators and approximation, Springer International Publishing., Switzerland.
- [6] Acar, T., Gupta, V. and Aral, A., 2011, Rate of convergence for generalized Szász operators, Bulletin of Mathematical Sciences 1(1), 99-113.
- [7] Ulusoy, G. and Acar, T., 2016, q-Voronovskaya type theorems for q-Baskakov operators. *Mathematical Methods in the Applied Sciences*, 39(12), 3391-3401.

### **Groupoid Structure on Homotopies of Simplicial Maps**

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key-words: Simplicial algebra, homotopy, groupoid.

### Abstract:

A simplicial (commutative) algebra  $\mathbf{X}$  is a collection of commutative k-algebras  $\{X_n\}$  together with the k-algebra homomorphisms  $d_i^n: X_n \to X_{n-1}, 0 \le i \le n$ ,  $(n \ne 0)$  and  $s_i^n: X_n \to X_{n+1}, 0 \le i \le n$  which are called face and degeneracy maps such that satisfying the usual simplicial identities given in [1]. A morphism of simplicial algebras  $f: \mathbf{X} \to \mathbf{Y}$  is a similicial map such that  $f_n: X_n \to Y_n$  is a morphism of algebras, for each n. Thus we get the category of simplicial algebras denoted by SimpAlg. In a simeplicial algebra, if we forget the dimensions higher than k, we obtain a k-truncated simplicial algebra, which is denotes by  $Tr_kSimpAlg$ .

A simplicial algebra X will be called free [2], if;

i)  $X_n$  is a free algebra with a given bases  $B_n$  for every integer  $n \ge 0$ ,

ii) The bases are stable under all degeneracy operators, i.e. for every pair of integers (i, n) with  $0 \le i \le n$  and every generator  $x \in X_n$ , the element  $s_i(x)$  is a generator of  $X_{n+1}$ .

In this work, we prove that the homotopy relation for free simplicial algebras maps  $\mathbf{X} \to \mathbf{X}'$  is an equivalence relation. Based on this, we construct a groupoid structure whose objects are the simplicial algebra maps between two fixed 1-truncated simplicial algebras (with free domain), and the morphisms are the homotopies between 1-truncated simplicial algebra maps.

- [1] Andrè, M., (1970): Homologie des Algebrès Commutatives, Springer-Verlag, 206
- [2] Mutlu A., Mutlu B. and Uslu E., (2009), To Construction of Free Simplicial Algebras with given CW-basis, *International Mathematical Forum*, 4 (30), 1489-1495

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### On a Modular Time Stepping Scheme of Velocity-Vorticity Method via Grad-Div Stabilization for the Navier-Stokes Equations

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key-words: Time stepping method, Velocity-vorticity formulation, the Navier-Stokes equations.

### Abstract:

This manuscript introduces a modular time filter scheme of velocity–vorticity (VV) method via grad-div stabilization for the incompressible Navier–Stokes equations (NSE). Velocity-vorticity method for NSE couples the rotation form of the momentum equation with the vorticity equation. The proposed numerical scheme adds two modular time filter steps for fluid variables, one is for the velocity and the other for the vorticity, into an existing backward-Euler finite element code for the VV-with grad-div stabilization. Time discretization is done in a way that it decouples velocity–pressure system with the vorticity equation at each time step and allows for simultaneous solving of the vorticity equation, velocity–pressure system and modular time steps. In the paper, it is shown that the proposed numerical scheme is well-posed and improves the accuracy in time from the first order to second order. In addition, the manuscript provides two numerical experiments. The first numerical experiment verifies theoretical convergence rates obtained from mathematical analysis. The second tests the algorithm on Gresho benchmark problem. In this experiment, the algorithm's solutions are compared with two steps backward difference method (BDF2) and modular time stepping scheme without grad-div stabilization. The results conclude that the solutions of the proposed algorithm yield much more accurate solutions.

- [1] Victor D., William L. and Haiyun Z., (2020), A Time-Accurate, Adaptive Discretization for Fluid Flow Problems, *International Journal of Numerical Analysis and Modeling*, 17(2), pp. 257-280.
- [2] Elenor J., Volker J., Alexander L. and Leo R., (2014), On the parameter choice in grad-div stabilization for incompressible flow problems, *Advances in Computational Mathematics*, 40(2), pp. 491-516.
- [3] Aytekin C., Fatma G. E. and Songul K., (2020), Analysis of Second Order Time Filtered Backward Euler Method for MHD Equations, *Journal o Scientific Computing*, 82, pp. 32.
- [4] Mine A., Alexander L., Leo R. and Philipp S., (2018), The analogue of grad-div stabilization in DG methods for incompressible flows: limiting behavior and extension to tensor-product meshes, *Computer Methods in Applied Mechanics and Engineering*, 341, pp. 917-938.

### The Inverse Problem of Discontinuous Sturm-Liouville Operator by Spectral Data

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key-words: Discontinuous Sturm-Liouville operator, inverse problem, spectral data.

### Abstract:

Consider the following discontinuous Sturm-Liouville boundary value problem

$$-y'' + q(x)y = \lambda^{2}\mu(x)y, \quad 0 < x < \pi,$$
  
$$y(a+0) = \beta y(a-0), \quad y'(a+0) = \beta^{-1}y'(a-0),$$
  
$$y'(0) = y'(\pi) + hy(\pi) = 0,$$

where  $q(x) \in L_2(0, \pi)$  is real valued function,  $\beta > 0$  and h are real constants,  $\lambda$  is a spectral parameter,  $\mu(x)$  is discontinuous coefficient:

$$\mu(x) = \begin{cases} 1, & 0 < x < a, \\ \alpha^2, & a < x < \pi, \end{cases}$$

with  $0 < \alpha \neq 1$ ,  $a \in (0, \pi)$ ,  $a > \frac{\alpha \pi}{\alpha + 1}$ . We study the inverse spectral problem of this problem that is indicated in the way: to determine the potential q(x) and the boundary constant h according to spectral data.

- [1] Akcay O., (2019): On the boundary value problem for discontinuous Sturm-Liouville, Mediterr. J. Math., 16(7), pp. 1-17
- [2] Akcay O., (2021): Uniqueness theorems for inverse problems of discontinuous Sturm-Liouville operator, Bull. Malays. Math. Sci. Soc., 44, pp. 1927-1940
- [3] Marchenko, V.A., (2011): Sturm-Liouville Operators and Applications, AMS Chelsea Publishing, Providence.

### Investigation of the Effects of Gender, Hypertension and Coronary Artery Disease (CAD) in Patients Diagnosed with COVID-19 by Categorical Data Analysis Approach

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key-words: Log-Linear model, categorical data analysis, hypertension, Coronary Artery Disease, COVID-19, three-way contingency table

### Abstract:

Log-linear analysis is commonly used to investigate relationships between more than two categorical variables when all relevant variables in the model are categorical. In the Log-linear model, relationships between the categorical variables can be determined by chi-square analysis based on observed frequencies and expected frequencies especially in three-way contingency tables. In this study, statistical significances of the main effects and possible interaction effects of the categorical variables of gender, hypertension (HT) and coronary artery disease (CAD) in patients diagnosed with the COVID-19 Pandemic have been evaluated by using the log-linear model approach. The main effects of gender, HT and CAD, and also gender\*HT, gender\*CAD, and CAD\*HT interaction effects have found to be statistically significant (p < 0, 05). So the importance of the gender, CAD and HT categorical variables on the COVID-19-diagnosed patients is emphasized.

- [1] Altun G., (2021), A Study On Covid-19 Data With Log-linear Model Approach, *Mugla Journal of Science and Technology*, 7(1), 52-58.
- [2] Chu, J., (2021), A statistical analysis of the novel coronavirus (COVID-19) in Italy and Spain, PloS one, 16(3), e0249037
- [3] Milewska A. J., Citko D., Jankowska D., Milewski R., Kononczuk K., Wiesak T., Morgan T., Milewski R., (2018) The Use of Log-linear Analysis for Pregnancy Prediction, *Studies in Logic, Grammar and Rhetoric* 56(69)
- [4] Odetunmibi O. A., Adejumo A. O., Anake T. A., (2019), Log-Linear Modelling of Effect of Age and Gender on the Spread of Hepatitis B Virus Infection in Lagos State, *Nigeria, Open Access Macedonian Journal of Medical Sciences*. 7(13):2204-2207.
- [5] Pandey, A., & Saxena, N. K., (2022), Effectiveness of government policies in controlling COVID-19 in India, *International Journal of Health Services*, 52(1), 30-37.

### **ROC Curve Analysis for the Measurements Distributed Power-Lindley Distribution**

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key-words: Area under the curve, Power-Lindley distribution, Receiver Operating Characteristic curve, Youden index .

### Abstract:

In this study, the ROC curve analysis is considered for the Power-Lindley distribution. The maximum likelihood estimates for the area under the curve and some cut-off points. Interval estimation is also discussed by using maximum likelihood and bootstrap methodologies. A Monte Carlo simulation is performed to observe the performance of the proposed methods. An illustrative example is provided.

- [1] Attwood K., Hou S., and Hutson A., (2022), Application of the skew exponential power distribution to ROC curves, *Journal of Applied Statistics, pp. 1-16.*
- [2] Gonçalves L., Subtil A., Oliveira M. R., and de Zea Bermudez P., (2014), ROC curve estimation: An overview, REVSTAT-Statistical Journal, vol. 12, pp. 1-20.
- [3] Yin J., Samawi H., and Tian L., (2022), Joint inference about the AUC and Youden index for paired biomarkers, *Statistics in Medicine*, vol. 41, pp. 37-64.

### Some Characterizations on Radical Lightlike Hypersurfaces of Almost Product-Like Lorentzian manifolds

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key-words: Lightlike hypersurface, statistical manifold, Lorentzian manifold.

### Abstract:

The main identities on radical invariant lightlike hypersurfaces of almost product-like Lorentzian manifolds are presented. With the help of statistical connections and Riemannian curvature tensors, some characterizations on radical invariant lightlike hypersurfaces of almost product-like statistical manifolds are obtained.

- Aksu Ö., Gülbahar M. and Erkan E., (2023), Lightlike hypersurfaces of almost product-like semi-Riemannian manifolds, Symmetry, 15(1), 77
- [2] Amari S., (1985): Differential-geometrical methods in statistics, Lecture Notes in Statistics, Springer-Verlag, 28
- [3] Duggal, K. L., Şahin, B. (2011): Differential geometry of lightlike submanifolds, Springer Science, Business Media: Berlin, Germany
- [4] Erkan E., Takano K. and Gülbahar M., (2022), Locally product-like statistical manifolds and their hypersurfaces, *arXiv:2212.02862*
- [5] Furuhata, H., Hasegawa, I. (2016): Submanifold theory in holomorphic statistical manifolds in: Geometry of Cauchy-Riemann submanifolds, Springer, Singapore
- [6] Kılıç E., Şahin B., (2008), Radical anti-invariant lightlike submanifolds of semi-Riemannian product manifolds, *Turk. J. Math.*, 32, 429-449
- [7] Takano K., (2010), Statistical manifolds with almost complex structures, Tensor, New Ser., 72, 225-231

### **On Toeplitz Matrices with Generalized Oresme Numbers**

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key-words: Oresme numbers, Toeplitz matrix, norm, Hadamard product, Kronecker product.

### Abstract:

This article is about Toeplitz matrices with Oresme numbers components. First, the Toeplitz matrices whose elements are the Oresme numbers are defined and then the Frobenius (Euclidian), row and column norms of these matrices are found. Furthermore lower and upper bounds are obtained for the spectral norms of these matrices. In addition, the upper bounds for the Frobenius and spectral norms of the Kronecker and Hadamard product matrices of the Toeplitz matrices with the Oresme numbers are calculated.

- [1] Akbulak, M., Bozkurt, D., On the norms of Toeplitz matrices involving Fibonacci and Lucas numbers, Hacettepe Journal of Mathematics and Statistics, 37(2), 89-95, 2008.
- [2] Daşdemir, A., On the norms of Toeplitz Matrices with the Pell, Pell-Lucas and Modfied Pell numbers, Journal of Engineering Technology and Applied Sciences, 1(2), 51-57, 2016.
- [3] Soykan, Y., Generalized Oresme Numbers, Eartline Journal of Mathematical Sciences, 7(2), 333-367, 2021.
- [4] Uygun, Ş., On the Bounds for the Norms of Toeplitz Matrices with the Jacobsthal and Jacobsthal-Lucas Numbers, Journal of Engineering Technology and Applied Sciences 4(3), 105-114, 2019.
## Simultaneous Approximations of Sampling Series on Weighted Spaces of Function and Image Processing with Digital Topology Methods

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key-words: Sampling series, simultaneous approximation, weighted function.

#### Abstract:

In this study, we have investigated the *m*-th order Kanatorovich type sampling series, alternatively referred to as the generalized sampling series. Our research focuses on establishing a set of theorems that demonstrate simultaneous convergence properties within weighted spaces of functions. Specifically, we have examined pointwise and uniform convergence, as well as the rate of convergence. Additionally, we have derived a Voronovskaja-type theorem to further contribute to the understanding of this topic. In the last part, we present some applications on image processing using the digital topology methods.

- [1] Acar, T., Costarelli, D. and Vinti, G., 2020, Linear prediction and simultaneous approximation by m-th order Kantorovich type sampling series, *Banach J. Math. Anal.* 14, 1481.
- [2] Sendov, B. and Popov, V., 1969 The convergence of the derivatives of positive linear operators., C. R. Acad. Bulgare Sci. (in Russion) 22, 507-509.
- [3] Gadjiev, A. D., 1976: Theorems of Korovkin type., Mathematical Notes of the Academy of Sciences of the USSR 20: 995-998.

## Solution of an Algebraic Linear System of Equations Using Fixed Point Results in $C^*$ -algebra Valued Extended Branciari $S_b$ -metric Spaces

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key-words:  $C^*$ -algebra,  $C^*$ -algebra valued Branciari  $S_b$ -metric space,  $C^*$ -algebra valued extended Branciari  $S_b$ -metric space.

#### Abstract:

Two new types of metric spaces namely  $C^*$ -algebra valued Branciari  $S_b$ -metric space and  $C^*$ -algebra valued extended Branciari  $S_b$ -metric space are introduced in this work, which are a generalization of all known related metric spaces. We explained the generalizations with two examples respectively. There are examples of self-mappings having a unique fixed point, which are concluded using our main theorems. As an application, we used one of our main results to show the existence of a unique solution of an algebraic system of linear equations with a numerical example.

- [1] Murphy, G. J., 1990: C\*-algebras and operator theory, Academic Press, London.
- [2] Ma, Z., Jiang, L. and Sun, H., 2014, C<sup>\*</sup> algebra valued metric spaces and related fixed point theorems, *Fixed Point Theory Appl*, 206.
- [3] Kamran, T., Samreen, M. and Ain, Q. UL., 2017, A generalization of *b*-metric space and some fixed point theorems, *Mathematics*, 5(2) 19.
- [4] Rohen, Y., Došenović, T. and Radenović, S., 2017, A note on the paper "A Fixed point Theorems in S<sub>b</sub>-Metric Spaces", *Filomat*, 31(11), 3335-3346.
- [5] Asim, M. and Imdad, M., 2022, C\*-Algebra valued extended b-metric spaces and fixed point results with an application, UPB Science Bulletin, Series A, 82(1), 207-218.
- [6] Alam, K. H., Rohen, Y. and Saleem, N., 2023, Fixed points of  $(\alpha, \beta, F^*)$  and  $(\alpha, \beta, F^*)$ -weak Geraghty contractions with an application, *Symmetry*, 15(1), 243, 1-17.
- [7] Mlaiki, N., 2018, Extended S<sub>b</sub>-metric spaces, J. Math. Anal., 9(1), 124-135.
- [8] Kalaivani, C. and Kalpana, G., 2018, Fixed point theorems in C\*-algebra valued S-metric spaces with some applications, Sci. Bull., Ser. A, Appl. Math. Phys., Politeh. Univ. Buchar., 80, 93-102.

### Estimation of Lifetime Performance Index for Power-Lindley Distribution Under Progressive First-Failure Censoring

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key-words: Lifetime performance index, Monte Carlo simulation, Power Lindley distribution, point estimation, progressively first-failure censoring.

#### Abstract:

The performance of lifetime is an important topic in the manufacturing industries, especially the employment of the lifetime performance index  $C_L$  that measures process capability and performance. In this paper, we study the inference on  $C_L$  index for power-Lindley distribution based on the progressive first-failure censored data which was introduced by Wu and Kuş (2009). We provide four different estimators of  $C_L$  namely, the maximum likelihood, Kolmogorov-Smirnov, Anderson-Darling and Cramérvon Mises types. A Monte Carlo simulation is conducted under different scenarios and we compare the estimators in terms of values of the mean squared error and bias. An illustrative example is provided with real data to compute the proposed estimates.

### References

Wu, S. J., & Kuş, C. (2009), On estimation based on progressive first-failure-censored sampling., *Computational Statistics & Data Analysis*, 53(10), 3659-3670

## **On Proximal Homotopy and Proximal Path Homotopy in Computational Proximity**

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key-words: Proximity, homotopy, path.

#### Abstract:

A (descriptive) proximity space is a non-empty set X together with a relation  $(\delta_{\Phi}) \delta$ , called a (descriptive) proximity relation, that satisfies some certain properties known as the (descriptive) proximal Čech axioms [2, 1]. Given a (descriptive) proximity space,  $((X, \delta_{\Phi})) (X, \delta)$ , Peters and Vergili introduced proximal homotopy between (descriptive) proximally continuous maps [3]. So, two (descriptive) proximally continuous maps are said to be (descriptive) proximal homotopic, provided there is a (descriptive) proximally continuous map that makes one map transform continuously during specific time into another map. The main goal of this study is to introduce some of the properties provided by this homotopy relation, and obtain some results that we think are useful for (descriptive) proximity spaces. In addition, we will introduce (descriptive) proximal path homotopy and (descriptive) proximal deformation. This study may lead to an important group structure for (descriptive) proximity spaces, which may come to be known as (descriptive) proximal fundamental group.

- Čech E., (1966): Topological Spaces, John Wiley and Sons Ltd., London, 1966, fr seminar, Brno, 1936-1939; rev. ed. Z. Frolik, M. Katetov.
- [2] Concilio Di A., Guadagni C., Peters J.F., and Ramanna S., (2018), Descriptive proximities. properties and interplay between classical proximities and overlap, *Math. Comput. Sci. vol. 12, no. 1, pp. 91–106.*
- [3] Peters J.F. and Vergili T., (2023), Good coverings of proximal Alexandrov spaces. Path cycles in the extension of the Mitsuishi-Yamaguchi good covering and Jordan Curve Theorems, *Applied General Topology*, vol.24, no.1, pp. 25-45.

## Some Results on Cartan Null and Pseudo Null Bertrand Curves in Minkowski 3-Space

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key-words: Bertrand curves, Cartan null curves, pseudo null curves, Minkowski 3-space.

#### Abstract:

In the theory of curves in Euclidean 3-space, it is well known that a curve  $\varphi$  with non-zero curvatures is said to be a Bertrand curve if for another curve  $\varphi^*$ , there exists a one-to-one correspondence between  $\varphi$  and  $\varphi^*$  such that both curves have common principle normal line. These curve have been studied in different space over a long period of time and found wide application in different areas. Therefore, we have a great knowledge of the geometric properties of these curves. In [2], the authours gave a new perspective to Bertrand curves. This point of view was also carried to curves in Minkowski 3-space [1, 3]. In [5], new results for spacelike Bertrand curves was obtained in the light of recent studies on Bertrand curves. In this talk, we give some results for Cartan null and pseudo null Bertrand curves in Minkowski 3-space by using this new perspective which generalize the notion of Bertrand curves in Euclidean 3-space. According to the this perspective, the necessary and sufficient conditions have been obtained for Cartan null and pseudo null curves to be Bertrand curves in Minkowski 3-space. In addition, some examples are given.

- [1] Uçum A. and İlarslan K., (2016) On Timelike Bertrand Curves in Minkowski 3-space. Honam Mathematical J. 38,3, 467-477.
- [2] Camei Ç., Uçum A. and İlarslan K., (2020) A New Approach to Bertrand Curves in Euclidean 3-Space. J. Geom. 111,49.
- [3] Altın Erdem H., Uçum A., Camcı Ç. and İlarslan K.,(2021) New approach to timelike Bertrand Curves in Minkowski 3-space. Accepted in Carpathian Mathematical Pub.
- [4] Altın Erdem H. and İlarslan K., (2022) Spacelike Bertrand Curves in Minkowski 3-space Revested. Accepted in Analele Stiintifice ale Universitatii, Ovidius Constanta.

## On Muitidimensional Urysohn Type Meyer-König and Zeller Operators

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key-words: Urysohn integral operators, Meyer-König and Zeller operators, approximation.

#### Abstract:

The basis of the theory of approximation of functions of a real variable is due to Weierstrass, it states that every continuous realvalued function f defined on [a, b] is approximatable by algebraic polynomials. Since the proof of this theorem was long and complicate, the proof of Weierstrass provoked many famous mathematicians to find simpler and more constructive proofs. In 1912, S.N. Bernstein [1] introduced the Bernstein polynomials to give the first constructive proof of the Weierstrass approximation theorem. And the basis of these polynomials is related with the Binomial distribution. Even if these operators are, very probably, the most studied linear positive operators and due to importance of the Bernstein polynomials, a variety of their generalizations and related problems have been studied. In 1960, Meyer-König and Zeller [2] gave some convergence results by the operators  $M_n(f, x)$ are defined as

$$(M_n f)(x) = \sum_{k=0}^{\infty} m_{n,k}(x) f\left(\frac{k}{n+k}\right)$$

$$(M_n f)(1) = f(1),$$

$$(1)$$

where  $m_{n,k}(x) = \binom{n+k-1}{k} x^k (1-x)^n$ ,  $x \in [0,1)$  is the well-known Meyer-Konig and Zeller basis which is related with

the geometric distribution. Clearly, integral equations have been of considerable significance in various fields of applied science and engineering. The most important and frequently investigated integral equation in nonlinear functional analysis are the Hammerstein and the Urysohn equations. The present study concerns the following Urysohn equation and the related Urysohn operator

$$Ux(t) = \int_{a}^{b} k(t, s, x(s))ds, \quad t \in [a, b]$$

where k is known and x is the unknown function to be determined. The goal of this study is generalization and extension of the theory of interpolation of functions to functionals and operators by a multidimensional Urysohn type Meyer-König and Zeller operators.

- [1] Bernstein, S. N., 1912, Demonstration du Théoreme de Weierstrass fondée sur le calcul des probabilités, *Comm. Soc. Math. Kharkow 13, pp. 1-2.*
- [2] Meyer-Konig, W. and Zeller, K., 1960, Bernsteinsche Potenzreihen, Studia Math., 19, pp. 89-94.
- Karsli, H., 2017, Approximation by Urysohn Type Meyer-König and Zeller Operators to Urysohn Integral Operators, *Results Math.*, 72, pp. 1571-1583.

## Hybrid Modeling Approaches for Biochemical Reaction Networks

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key-words: Modeling, biochemical reaction networks.

#### Abstract:

Biochemical reaction networks (BRNs) are a set of species that interacts with each other via different reaction channels. The deterministic and stochastic approaches are two fundamental approaches to modeling these systems. The deterministic approach considers the time evolution of BRNs as continuous, deterministic and uses Reaction Rate Equations (RREs) to model these processes. By contrast, the stochastic approach models these systems using discrete, integer-valued Markov processes and utilizes the Random Time Change Model (RTCM) to explain their dynamics. In the stochastic approach, the system's probability density function satisfies the Chemical Master Equation (CME). Although these approaches seem very different, the diffusion approximation representing the state of the BRNs using the Chemical Langevin Equation (CLE) constructs a bridge between the CME and the RRE. In diffusion approximation, the probability density function of BRNs satisfies the Fokker-Planck equation (FPE) [3].

In general, BRNs have a multi-scale nature in terms of reaction rates and abundance of species. Therefore, pure traditional approaches such as deterministic and stochastic approaches fail to explain the dynamics of these multi-scale BRNs. This fact leads to hybrid models that combine different modeling approaches to explain the dynamics of these type of BRNs. In [2], we proposed jump-diffusion approximation to model the dynamics of BRNs with multi-scale nature. The first step of the method is to partition the reactions into fast and slow subgroups. In the second step of the method, the fast reactions are modeled using diffusion approximation, while the Markov chain representation is kept for slow reactions. As a result, the system's state vector is represented as a combination of the RRE and the CLE. In [1], we proved that the joint probability density function of the jump-diffusion approximation satisfies the hybrid master equation (HME), which involves terms from the CME and the FPE, and presented an algorithm to approximate its solution. In this talk, based on the studies [2] and [1], we give an overview of the hybrid modeling approaches for the BRNs. Furthermore, we assess the validity of the proposed methods using BRNs of different sizes.

- D. Altıntan and H. Koeppl (2020): Hybrid Master Equation for Jump-Diffusion Approximation of Biomolecular Reaction Networks, *BIT Numerical Mathematics*, vol.60, pp. 261–294
- [2] A. Ganguly, D. Altıntan and H. Koeppl (2015): Jump-Diffusion Approximation of Stochastic Reaction Dynamics: Error Bounds and Algorithms, *Multiscale Model. Simul.*, vol. 13, pp. 1390–1419
- [3] D. J. Wilkinson, (2006): Stochastic modelling for systems biology, Boca Raton, FL : Taylor & Francis.

## Majorization by Starlike functions with respect to t-symmetric points of complex order

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key-words: Analytic function, Starlike function, Convex function, Symmetric points, complex order, majorization.

#### Abstract:

Let denote  $STS(t, \beta, \gamma)$  the class of starlike functions with respect to t-symmetric points of complex order which are analytic in the open unit disk  $U = \{z \in \mathbb{C} : |z| < 1\}$  and satisfy the following condition

$$\left|\frac{1}{\gamma}\left(\frac{2zf'\left(z\right)}{f\left(z\right)-tf\left(-z\right)}-1\right)\right| < \beta$$

where  $z \in \mathcal{U}$ ,  $-1 < t \leq 1$ ,  $0 < \beta \leq 1$  and  $\gamma \in \mathbb{C}$ ,  $\gamma \neq 0$ .

If f(z) is majorized by  $g(z) \in STS(t, \beta, \gamma)$ , then we obtain the radius for  $|f'(z)| \leq |g'(z)|$  in Theorem and corollaries of this theorem.

#### References

[1] Altıntaş O., Owa S., (1992), Majorizations and Quasi.Subordinations for Certain Analytic Functions, *Proc. Japan Acad., vol.* 68, Ser. A, pp. 181-185

## **Constructions of some minimal exotic 4-manifolds**

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key-words:Mapping Class Group, Symplectic 4-manifolds, Lefschetz fibrations.

#### Abstract:

Lefschetz fibrations have one to one correspondence with the relations in mapping class groups and they are very useful tool to construct exotic (homeomorphic but not diffeomorphic) 4-manifolds. In this talk, I will discuss a few results about Lefschetz fibrations which are used to construct some exotic 4-manifolds.

## New Approach for Fractional Differential Equations of Variable Order

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key-words: Derivatives and integrals of variable-order, fixed point theorem, Cauchy-type problem, Ulam-Hyers stability.

#### Abstract:

In this paper, we use a novel approach to study the existence, uniqueness, and stability of solutions to a Cauchy-type problem with fractional variable order. Contrary to the techniques taken in the literature, which were centered on the usage of the concept of generalized intervals and the idea of piecewise constant functions, our approach is straightforward and based on a novel fractional operator that is more appropriate and demonstrate the solvability and stability of the main problem under less restrictive presumptions. The application, which includes an example and supporting images, concludes the paper.

- [1] H. Khan, J. Alzabut, H. Gulzar, (2023), Existence of solutions for hybrid modified ABC-fractional differential equations with p-Laplacian operator and an application to a waterborne disease model, *Alexandria Engineering Journal*, *70*, 665-672.
- [2] A. Benkerrouche, M.S. Souid, S. Chandok, A. Hakem, (2021): Existence and stability of a Caputo variable-order boundary value problem, *Mathematics Article ID* 7967880, 1-16.
- [3] S. Zhang, (2013): Existence and uniqueness result of solutions to initial value problems of fractional differential equations of variable-order, *Journal of Fractional Calculus and Applications*, 4(1), pp. 82-98.

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### **Difference Sequence Space of Fractional Order Generated by Bell Numbers**

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numbers, difference operator, matrix transformations.

#### Abstract:

In this paper, we introduce Bell difference sequence spaces of fractional order  $\alpha$ ,  $\ell_p(B^{(\alpha)})$  defined by the composition of the fractional order difference operator  $\Delta^{(\alpha)}$  and the Bell matrix  $\tilde{B} = (\tilde{b}_{ij})$ . We give some topological properties, Schauder basis and  $\alpha$ -,  $\beta$ -,  $\gamma$ -duals of the newly defined spaces. Finally, we examine certain classes of compact operators on  $\ell_p(B^{(\alpha)})$  using Hausdorff measure of non-compactness.

- [1] Karakas, M., 2022, On the sequence spaces involving Bell numbers, Linear and Multilinear Algebra, https://doi.org/10.1080/03081087.2022.2098225.
- [2] Yaying, T., Hazarika, B. and Mohiuddine, S. A., 2021, On difference sequence spaces of fractional order involving Padovan numbers, Asian-European Journal of Mathematics, Vol:14, No:6, 1-24.
- [3] Baliarsingh, P., 2013, Some new difference sequence spaces of fractional order and their dual spaces, Applied Mathematics and Computation, Vol:219, 9737-9742.
- [4] Baliarsingh, P. and Dutta, S., 2015, A unifying approach to the difference operators and their applications, Bol. Soc. Paran. Mat., Vol:33, 45-57.

### An Inverse Problem For a Nonlinear Fractional Diffusion Equation

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key-words: Nonlinear fractional diffusion equation, initial-boundary value problem, inverse problem.

#### Abstract:

In this work, we consider a nonlinear fractional diffusion equation with initial and boundary conditions on a bounded domain. Using the theory given in [1], inverse problems for linear fractional diffusion equations were studied by [4]. The solution of a direct problem for nonlinear fractional diffusion equation was obtained by [3] and [2] for Neumann and Dirichlet boundary conditions, respectively. In light of these works, we aim to investigate the conditional stability in an inverse source problem of determining t-dependent factor in the source term by observation at one point over (0,T) for T>0. In order to prove our main result, we estimate the formal solution which is obtained by the spectral theory and Laplace transform.

- [1] Brezis H., (2011): Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.
- [2] Jin B., (2021): Fractional Differential Equations, An Approach via Fractional Derivatives, Springer Nature.
- [3] Luchko Y., Rundell W., Yamamoto M. and Zuo L., (2013), Uniqueness and Reconstruction of an Unknown Semilinear Term in a Time-Fractional Reaction–Diffusion Equation, *Inverse Problems, vol. 29, pp. 1-16.*
- [4] Sakamoto K. and Yamamoto M., (2011), Initial Value/Boundary Value Problems for Fractional Diffusion-Wave Equations and Applications to Some Inverse Problems, *Journal of Mathematical Analysis and Applications, vol. 382, pp. 426-447.*

### New Results on Timelike Mannheim Curves in Minkowski 3-Space

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key-words: Mannheim curves, timelike curves, Minkowski 3-space.

#### Abstract:

It is well known that a curve  $\beta : IBE^3$  with non-zero curvatures is said to be a Mannheim curve if there is a curve  $\beta^* : I^*BE^3$  such that the principal normal vectors of  $\beta(s)$  coincide with the binormal vectors of  $\beta^*(s^*)$  at  $s \in I$ ,  $s^* \in I^*$ , in the theory of curves in Euclidean 3-space. These curve have been studied in dis space over a long period of time and found wide application in unlike areas. Therefore, we have a great knowledge of the geometric properties of these curves. In [1], Mannheim partner curves were studied 3-dimensional space. This point of view was also carried to curves in Minkowski 3-space [2, 3]. In [5], the authours dedicated a new approach to Mannheim curves. In [5], the authours gave a new approach to Mannheim curves in Minkowski 3-space. Thanks to this new approach, new Mannheim curve examples, which are not known in the literature, are obtained and new results are given. In this talk, timelike Mannheim curves are reconsidered from the perspective of this new approach. The new results obtained were supported with examples.

- [1] Liu H., Wang, F., Mannheim partner curves in 3-space. J. Geom. 88 (2008), no. 1-2, 120–126.
- [2] Akyigit, M., Ersoy, S., Özgür, İ., and Tosun, M., Generalized timelike Mannheim curves in Minkowski space-time E41. Math. Probl. Eng. 2011, Art. ID 539378, 19 pp.
- [3] Grbović, M., İlarslan, K., and Nešović, E., On null and pseudo null Mannheim curves in Minkowski 3-space. J. Geom. 105 (2014), no. 1, 177–183.
- [4] Camcı, Ç., Uçum, A. and İlarslan, K., A New Approach to Mannheim Curve in Euclidean 3-Space, Tamkang Journal of Mathematics, (Accepted) 54, (2023).
- [5] Arıcı H. İ., İlarslan, K., A New Approach to Spacelike Mannheim Curve in Minkowski 3-Space, Submited (2023).

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## Fuzzy Logic Based Optimization of Multivariate Nonlinear Regression Models and Applying

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key-words: mathematical modeling, symbolic computation, fuzzy logic, optimization algorithms .

#### Abstract:

Polynomial regression is one of the most used methods to model relationships between data in software algorithms. In many cases, linear regression using a single input variable may not provide sufficient predictive power in models related to daily life events. This situation has led researchers to the multiple nonlinear regression technique, in which two or more variables are used simultaneously as inputs for the regression model. The use of this mathematical model alone or without optimization in software algorithms causes the desired success in regression applications to not be achieved. For some datasets, it may be necessary to increase the degree of polynomial in regression applications. However, as the degree of polynomial increases, the model becomes more complex and there is a risk of overfitting. This situation reduces the usability in software algorithms and prolongs the processing time. With the fuzzy logic-based optimization algorithm we developed for this process, the optimum polynomial degree and terms for the polynomial function are available. In this way, software using these mathematical models has been provided to be more successful and faster.

- [1] Basak, J.K., Qasim, W., Okyere, F.G., Khan, F., Lee, Y.J., Park, J., Kim, H.T., (2019), Regression Analysis to Estimate Morphology Parameters of Pepper Plant in a Controlled Greenhouse System, J. Biosyst. Eng., vol. 44, pp. 57-68
- [2] Boldina, I., Beninger, P.G., (2016), Strengthening statistical usage in marine ecology: linear regression, J. Exp. Mar. Bio. Ecol., vol. 474, pp. 81-91
- [3] Panigrahi, N., Das, B.S., (2021), Evaluation of regression algorithms for estimating leaf area index and canopy water content from water stressed rice canopy reflectance, *Inf. Process. Agric., vol. 8, pp. 284-298*
- [4] Yu, P., Low, M.Y., Zhou, W., (2018), Design of experiments and regression modelling in food flavour and sensory analysis: a review, *Trends Food Sci. Technol.*, vol. 71, pp. 202-215
- [5] Fattahi, H., Babanouri, N., (2017), Applying optimized support vector regression models for prediction of tunnel boring machine performance, *Geotechnical and Geological Engineering*, vol. 35, pp. 2205-2217
- [6] Strijov, V., Weber, G. W., (2010), Nonlinear regression model generation using hyperparameter optimization, Computers and Mathematics with Applications, vol. 60, pp. 981-988

## A New Approach to MacMahon's Equidistribution Theorem

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key-words: Permutation statistics, inversion number, major index.

#### Abstract:

Let  $S_n$  be a symmetric group. In [1], MacMahon proved that inversion number and major index are equi-distributed over the group  $S_n$ . That is, a well-known classical result due to MacMahon asserts that

$$\sum_{\sigma \in S_n} q^{inv(\sigma)} = \sum_{\sigma \in S_n} q^{maj(\sigma)} = \prod_{i=1}^n [i]_q \tag{2}$$

where q is indeterminate and  $[i]_q := \frac{1-q^i}{1-q}$  for each  $i, i = 1, \dots, n$ . The first bijective proof of this equi-distribution in the equation 7 was given by Foata in [2]. Second bijective proof, essentially due to Carlitz in [3], is sometimes known as *insertion method*. We will provide a new bijective approach to the equi-distribution in the equation 7 by using the inversion table concept and a different set of generators for  $S_n$  together.

- [1] MacMahon P. A., (1915), Combinatory analysis, 1, Cambridge University Press, London
- [2] Foata D., (1968), On the Netto inversion number of a sequence, Proc. Amer. Math. Soc., 19(1), 236-240
- [3] Carlitz L., (1975), A combinatorial property of q-Eulerian numbers, Amer. Math. Monthly, 82, 51-54.

## Generalized Fourier Transform: Illustrative Examples and Applications to Differential Equations

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key-words: Fourier transform, fractional derivatives and integrals, ordinary differential equations, fractional differential equations, special functions.

#### Abstract:

In this paper, we introduce a new generalized Fourier transform with an h-exponential function at its kernel. Then we give some properties of this new generalized Fourier transform. Furthermore, we compute the new generalized Fourier transforms of some generalized gamma and beta functions and fractional operators with special choices of the h-exponential function. Finally, with the help of this new generalized Fourier transform, we obtain the solutions of the differential equations of ordinary electric current and fractional motion.

- [1] Andrews G.E., Askey R. and Roy R., (1999): Special Functions, Cambridge University Press, Cambridge.
- [2] Bracewell R.N., (2000): The Fourier Transform and Its Applications, McGraw-Hill Higher Education, Third Edition.
- [3] Debnath L. and Bhatta D., (2015): Integral Transforms and Their Applications, Third Edition, CRC Press, New York.
- [4] Kilbas A.A., Srivastava H.M. and Trujillo J.J., (2006): *Theory and Applications of Fractional Differential*, North-Holland Mathematics Studies 204.
- [5] Samko S.G., Kilbas A.A. and Marichev O.I., (1993): *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach Science Publ., New York, London.
- [6] Stein E.M. and Shakarchi R., (2007): Fourier Analysis: An Introduction (Princeton Lectures in Analysis I), Princeton, Princeton University Press.

## Generalized Laplace Transform: Illustrative Examples and Applications to Differential Equations

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key-words: Laplace transform, fractional derivatives and integrals, fractional differential equations, special functions.

#### Abstract:

In this paper, we introduce a new generalized Laplace transform with an h-exponential function at its kernel. Then we give some properties of this new generalized Laplace transform. Furthermore, we compute the new generalized Laplace transforms of some generalized gamma and beta functions and fractional operators with special choices of the h-exponential function. Finally, with the help of this new generalized Fourier transform, we obtain the solutions of the differential equations of fractional Bagley-Torvik and fractional harmonic vibration.

- [1] Andrews G.E., Askey R. and Roy R., (1999): Special Functions, Cambridge University Press, Cambridge.
- [2] Debnath L. and Bhatta D., (2015): Integral Transforms and Their Applications, Third Edition, CRC Press, New York.
- [3] Jafari H., (2021), A new general integral transform for solving integral equations, *Journal of Advanced Research, vol. 32, pp.* 133-138.
- [4] Kilbas A.A., Srivastava H.M. and Trujillo J.J., (2006): *Theory and Applications of Fractional Differential*, North-Holland Mathematics Studies 204.
- [5] Watugala G.K., (1993), Sumudu transform: a new integral transform to solve differential equations and control engineering problems, *Integrated Education*, vol. 24, pp. 35-43.

## Electromagnetic Plane Wave Scattering from Perfect Electric Conducting Triaxial Ellipsoid

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key-words: Plane wave scattering, perfect electric conductors, triaxial ellipsoid.

#### Abstract:

In the present work, scattering of electromagnetic plane waves from perfect electric conducting triaxial ellipsoids are considered by treating them as a perturbed sphere and utilizing the results of [1] where electromagnetic scattering problems for arbitrarily shaped obstacles are formulated analytically. The aim of the present work is to detail the results of [1] for different size and orientations of ellipsoids and discuss the effects of these changes on both backward and forward radar cross sections. The obtained results are compared with a full wave solver to discuss limitations and validity of the results.

### References

[1] B. Ates, A. Kustepeli and Z. Cetin, (2021), Analytical Improvement on the Electromagnetic Scattering From Deformed Spherical Conducting Objects", *IEEE Transactions on Antennas and Propagation, vol.69 (12), pp:8630-8640.* 

### Some Characterizations of Associated Curves in Minkowski 3-Space

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key-words: Associated curve, Minkowski space, helix, slant helix.

#### Abstract:

In this study, we firstly define a *D*-direction and a *c*-direction curve of a non-null curve via the alternative frame  $\{n, c, w\}$  in Minkowski 3-space. Then we present some relationships by using some non-null helical curves. Furthermore, we introduce an alternative frame along a null curve which has no null vector. By using this alternative frame, we study direction curves of a null curve and characterize them. Also, we support the theory with illustrated examples.

- [1] Uzunoglu B., Gok I., Yayli Y., (2016), A new approach on curves of constant precession, Appl. Math. Comput., 275, 317-323
- [2] Choi J., and Kim Y., (2012), Associated curves of a frenet curve and their applications, Appl. Math. Comput., 2018, 9116-9124
- [3] Izumiya S. and Takeuchi N., (2004), New special curves and developable surfaces, Turk. J. Math., 28, 153-163
- [4] Macit N. and Duldul M., (2014), Some new associated curves of a Frenet curve in  $E^3$  and  $E^4$ , Turk. J. Math., 38, 1023-1037
- [5] Scofield P.D., (1995), Curves of Constant Precession, Am. Math. Mon., 102(6), 531-537
- [6] Ozdamar E., Hacisalihoglu H.H., (1975), A characterization of inclined curves in Euclidean n-space, Communications de la Faculté des Sciences de l'Université d'Ankara, 24, 15-22

### **On Maximal Zagreb Index**

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key-words: Graph theory, maximal index, topological index, Zagreb index.

#### Abstract:

Zagreb index, one of the most important topological index, was first considered in 1970s. In this paper, a new graph invariant, named maximal Zagreb index, is introduced. Also lists on equality results over special graphs with respect to maximal Zagreb index are given as well as new bounds on general graphs. Moreover, the extremal maximal Zagreb indices among unicyclic, bicyclic and acyclic graphs have been found by using some graph transformations.

- Das K. C., Yurttas A., Togan M., Cevik A. S. and Cangül I. N., (2013), The multiplicative Zagreb indices of graph operations, Journal of Inequalities and Applications, vol. 2013:90.
- [2] Deng H., (2007), A Unified Approach to the Extremal Zagreb Indices for Trees, Unicyclic Graphs and Bicyclic Graphs, MATCH Communications in Mathemetical and in Computer Chemistry, vol. 57, pp. 597-616.
- [3] Gutman I. and Das K. C., (2004), The first Zagreb index after 30 years, *MATCH Communications in Mathemetical and in Computer Chemistry, vol. 50, pp. 83-92.*

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On the Ramanujan-Nagell Equation  $x^2 + 3^a .11^b .17^c = 2^{\delta} .y^n$ 

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key-words: Diophantine equation, Ramanujan-Nagell type equation, Lehmer sequence.

#### Abstract:

In this paper, we completely solve the Diophantine equation  $x^2 + 3^a \cdot 11^b \cdot 17^c = 2^r \cdot y^n$  in nonnegative integers  $x, y \ge 1$ ,  $a, b, c, r \ge 0$  and  $n \ge 3$  with gcd(x, y) = 1. The proof depends on the deep result of Bilu, Hanrot and Voutier on the existence of primitive divisors in Lehmer sequences together with some basic properties of rings of algebraic integers.

- [1] Alan M. and Zengin U., (2020), On the Diophantine equation  $x^2 + 3^a 41^b = y^n$ , Periodica Mathematica Hungarica, 81, 284-291
- [2] Bhatter S., Hoque A. and Sharma R.,(2019), On the solutions of a Lebesgue-Nagell type equation, *Acta Mathematica Hungarica*, 158, 17-26
- [3] Bilu Y., Hanrot G. and Voutier F.M., (2001), Existence of primitive divisors of Lucas and Lehmer numbers (with an appendix by M. Mignotte), *Journal fur die Reine und Angewandte Mathematik*, 539, 75-122
- [4] Gou S. and Wang T.T., (2012), The Diophantine equation  $x^2 + 2^a 17^b = y^n$ , Czechoslovak Mathematical Journal, 62, 645-654
- [5] Le M., (1993), On the Diophantine equations  $d_1x^2 + 2^{2m}d_2 = y^n$  and  $d_1x^2 + d_2 = 4y^n$ , Proceedings of the American Mathematical Society, 118, 67-70

## **Unbounded Order Convergence Properties of Operator Nets**

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key-words: Vector lattices, unbounded order convergence, operators nets.

#### Abstract:

In this talk, firstly we will start with some preliminary notes about vector lattices. Then we will introduce unbounded order convergence properties on a completely regular Hausdorff space. Lastly, the characterization of uo-convergence on a Tychonoff space will be given by the means of operator nets.

### References

[1] Nieberg, P. M., 1991: Banach Lattices, Springer Verlag.

- [2] Aliprantis, C. D. and Burkinshaw, O., 2003: Locally Solid Riesz Spaces with Applications to Economics, *Mathematical Surveys* and *Monographs*, American Mathematical Society, vol 105.
- [3] Bilokopytov, E. and Troitsky, V. G., 2022: Order and uo-convergence in spaces of continuous functions, *Topology and its Applications*, vol 308.
- [4] Bilokopytov, E., 2021, Order continuity and regularity on vector lattices and on lattices of continuous functions, arXivpreprint:2103.08776v2.

### The Shortest Path is not always the Fastest

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key-words: The Brachistocrone curve, Euler-Lagrange equation, calculus of variations.

**Abstract:** In this talk, we will discuss about finding the shape of the curve down which a bead sliding from rest and accelerated by gravity assuming that there is no friction will slip from one point to another in the least time. The answer to this question is the Brachistocrone curve. The term derives from the Greek "brachistos" <u>the shortest</u> and "chronos" <u>time</u>. The Brachistocrone problem was one of the earliest problems posed in calculus of variations. It is said that Galileo (1564-1642) first presented this problem. It is also known that the cycloid is the curve which yields the quickest descent which may be handled under the field known as the calculus of variations, or variational calculus in physics, [1]. There are several solutions to this problem, one of which is a direct application of the Euler-Lagrange differential equation. All solutions are variational in nature and have one or another connection to the calculus of variations, [2].

- [1] Nishiyama Y., (2011), The Brachistochrone Curve: The Problem of Quickest Descent, Osaka Keidai Ronshu, 61(6), 309-316
- [2] Grebnev H.R, (2018): The Calculus of Variations and the Variational Differential Geometry, https://sites.math.washington.edu/ hgrebnev/.

### Some New Properties of Line Graphs and Their Omega Invariants

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key-words: Omega invariant, line graph, number of regions, triangular number.

#### Abstract:

For simple connected graphs having a fixed number of vertices, we have lower and upper bounds for the number of edges and also for some other graph parameters. The line graph of a graph is obtained from the graph by adding a new vertex onto each edge of the given graph and then joining them if the corresponding edges are adjacent to each other. Line graph is a derived graph and has many important applications in Chemistry. Omega invariant is a recently defined topological and combinatorial graph invariant. In this study, we deal with the line graphs of graphs and obtain several general properties of them. Especially, the omega invariant and the number of edges of the line graphs are obtained. Several relations between some graph parameters of a graph and its line graph are obtained. The results given here have many combinatorial properties and applications.

- [1] Delen S., Cangul I.N., (2019), Extremal Problems on Components and Loops in Graphs, Acta Mathematica Sinica, English Series, 35(2), 161-171
- [2] Delen S., Cangul I.N., (2018), A New Graph Invariant, Turkish Journal of Analysis and Number Theory, 6(1), 30-33
- [3] Ozden H., Ersoy Zihni F., Ozen Erdogan F., Cangul I.N., Srivastava G., Srivastava H.M., (2020), Independence Number of Graphs and Line Graphs of Trees by Means of Omega Invariant, *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas, 114:91*

## On Kantorovich Variant of Brass-Stancu Operators

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key-words: Brass-Stancu-Kantorovich operators,  $L^p$ -convergence, averaged modulus of smoothness, variation detracting property.

#### Abstract:

In this study, we deal with Kantorovich type generalization of the Brass-Stancu operators. For the sequence of these operators, we study  $L^p$ -convergence, and give some upper estimates for the  $L^p$ -norm of the approximation error via first-order averaged modulus of smoothness and the first-order K-functional. Moreover, we show that the Kantorovich generalization of each Brass-Stancu operator satisfies variation detracting property and is bounded with respect to the norm of the space of functions of bounded variation on [0, 1].

### References

[1] Brass H., (1971), Eine Verallgemeinerung der Bernsteinschen Operatoren, Abh. Math. Sem. Univ. Hamburg, vol. 36, pp. 111-122.

[2] Stancu D., (1983), Approximation of functions by means of a new generalized Bernstein operator, Calcolo, vol. 20(2), 211-229.

### Mathematical Analysis of Ionospheric Anomalies Before and After the Mw 7.8, Southeastern Turkey Earthquake

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key-words: Total electron content (TEC); solar activity (SA); geomagnetic storm (GS); February 6, 2023, Southeastern Turkey earthquake .

#### Abstract:

Southeastern Turkey perished by the Mw 7.8, Kahramanmaraş earthquake on February 6, 2023, at 01:17 UT (04:17 LT). The Kahramanmaraş-centered earthquake, hit by the break of the East Anatolian Fault Line, is felt in Syria, Egypt, Lebanon, Iraq, and Cyprus on the border of Turkey. Together with Syria, it causes loss of lives and severe damage in 10 cities. This work is dedicated to the people of the region. It discusses the causality of anomalies by conducting ionospheric anomaly research over the TEC values obtained from the CODE data center, which produces a global TEC map from GNSS data according to the epicenter coordinates (latitude and longitude) determined by the National seismology center Boğaziçi University Kandilli Observatory and Earthquake Research Institute. The TEC amplitudes are observed by evolving the time-domain TEC map into the frequency-domain with the Fourier transform. The peaks of the amplitudes allow preliminary information for the anomaly days. For the anomaly, the TEC (TECU) boundaries are drawn by the statistical specification. The TEC map outside these boundaries is marked as an anomaly. The causality of abnormalities is tried to be read through the triad of geomagnetic storm(s), solar activity, and the earthquake effect. One can find seven anomaly days in the paper. January 30, February 2, 3, 11, 15, 16, and 21 are the anomaly days. The essay probably detects the anomaly before 3 days related to the Southeastern earthquake.

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### Analysis of COVID-19 Dynamics in Turkey using Sinusoidal Transmission Rates and Stochastic Differential Equations

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key-words: Covid-19, stochastic noise, sinusoidal wave, simulation.

#### Abstract:

In this study, a deterministic compartmental model of COVID-19 transmission is used to investigate the dynamics of the disease in Turkey. The parameters related to the spread of the virus are defined as sinusoidal functions with specific amplitudes and frequencies to model the several waves of COVID-19 disease experienced around the world. Stochastic noise is added to each equation of the system to obtain a stochastic differential equation system for modelling the volatility in the spread of the disease. Turkish demographic data and statistics released by Turkish Ministry of Health will be used to define certain components of the system to estimate possible scenarios for Turkey from the results of the simulations.

- [1] Hassan, M. N., Mahmud, M. S., Nipa, K. F. and Kamrujjaman, M., (2023), Mathematical Modeling and Covid-19 Forecast in Texas, USA: a prediction model analysis and the probability of disease outbreak, *Disaster medicine and public health preparedness*, *17*, *e19*.
- [2] Kloeden, P. E. and Platen, E., (1992), Numerical Solution of Stochastic Differential Equations, Springer Berlin Heidelberg.
- [3] Khajanchi, S., Sarkar, K. and Banerjee, S., (2022), Modeling the dynamics of COVID-19 pandemic with implementation of intervention strategies, *The European Physical Journal Plus*, *137*(1), *129*.
- [4] Turkish Ministry of Health, 2023, COVID-19 Information Platform, https://covid19.saglik.gov.tr/, Access Date: 25/5/2023.
- [5] Awasthi, A., (2023), A mathematical model for transmission dynamics of COVID-19 infection, *The European Physical Journal Plus*, 138(3), 285.
- [6] Intarapanya, T., Suratanee, A., Pattaradilokrat, S. and Plaimas, K., (2023), Modeling the Spread of COVID-19 with the Control of Mixed Vaccine Types during the Pandemic in Thailand, *Tropical Medicine and Infectious Disease*, 8(3), 175.

### **Second Type Parametric Unified Apostol-Bernoulli Type Polynomials**

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key-words: Second type parametric unified Apostol-Bernoulli type polynomials, second type unified Apostol-Bernoulli type polynomials, generalized Stirling numbers of the second kind.

#### Abstract:

In this paper, we inspire by some generalizations applied to classical Bernoulli, Euler, and Genocchi polynomials. We apply some generalizations to type 2 Bernoulli, Euler, and Genocchi polynomials. Then we define these generalizations with a single polynomial expression. Finally, we give some theorems and relations that these polynomials provide.

- [1] Apostol T. M., (1998), Introduction to analytic number theory. Springer Science and Business Media.
- [2] Bilgic S., (2023): On the second type unified Apostol-Bernoulli type polynomials. *Proceedings Book of GFSNP 2023*, Antalya, pp. 256-261, 978-625-00-1128-7.
- [3] Cam S., (2005), Stirling Sayıları. Matematik Dünyası Dergisi Bahar, 30-34.
- [4] Cheon G. S., (2003), A note on the Bernoulli and Euler polynomials. Applied Mathematics Letters, 16(3), 365-368.
- [5] Comtet L., (1974), Advanced Combinatorics, Dordrecht: Reidel.
- [6] Hsu L. C. and Shiue P. J. S., (1998), A unified approach to generalized Stirling numbers. Advances in Applied Mathematics, 20(3), 366-384.
- [7] Ozden H., Simsek Y. and Srivastava H. M., (2010), A unified presentation of the generating functions of the generalized Bernoulli, Euler, and Genocchi polynomials. *Computers and Mathematics with Applications*, 60(10), 2779-2787.
- [8] Srivastava H. M., Masjed-Jamei M. and Beyki M. R., (2018), A parametric type of the Apostol-Bernoulli, Apostol-Euler and Apostol-Genocchi polynomials. Appl. Math. Inf. Sci, 12(5), 907-916.

### Application of Exponential Smoothing and Auto Regressive Integrated Moving Average (ARIMA) Mathematical Models to Radon Time Series Analysis

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key-words: Artificial intelligence, machine learning, mathematical modelling.

#### Abstract:

In this study, Exponential Smoothing and Auto Regressive Integrated Moving Average (ARIMA) mathematical modelling methods were applied on Radon (Rn) gas time series analysis and statistically significant predictions were obtained on earthquake predictions. In the research, 1250 Rn data belonging to the years 2007-2010 on the North East Anatolian Fault Zone (NEAFZ) were used and 1000 of these data were used for the establishment of the modelling and 250 for the testing of the modelling. Winter Additive Method was used during the test procedures. It was concluded that new models, time series and algorithms can be obtained by obtaining Rn data for unknown (Rn data not available) time periods in prediction studies.

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## **On Jain Operators**

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key-words: Positive linear operators, Jain operators.

#### Abstract:

A class of operators introduced by G. C. Jain in the 1970s is of great importance without a doubt. This talk aims to collect some published results on Jain and Jain-type operators and make a quick overview without an exhaustive treatment.

#### References

[1] Jain, G. C., 1972, Approximation of functions by a new class of linear operators, J. Aust. Math. Soc. J., (13) 3, 271-276.

[2] Agratini, O., 2018, A stop over Jain operators and their generalizations, An. Univ. Vest Timiş. Ser. Mat. Inform., 2, 28-42.

## A New Generalization of Kantorovich Operators Depending on a Non-negative Integer

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key-words: Kantorovich operators, Stancu operators, Korovkin's theorem,  $L^p$ -convergence, modulus of continuity.

#### Abstract:

In the present paper, we define Kantorovich variant of generalized Stancu operators depending on a non-negative integer parameter. We obtain approximation theorems in the space of continuous functions and  $L^p$ -space. Also we establish some estimates for the rate of convergence by using modulus of continuity and integral modulus of continuity.

- [1] Barbosu D., (2004), Kantorovich-Stancu type operators, *JIPAM (Journal of Inequalities in Pure and Applied Mathematics)*, 5 (3), 1-6.
- [2] Bostanci T. and Başcanbaz-Tunca G. (2022), On Stancu Operators Depending on a Non-Negative Integer, *Filomat 36:18*, 6129–6138.
- [3] Kantorovich L. V., (1930), Sur certains d'eveloppements suivant les polyn^omes de la forme de S. Bernstein, I, II, C. R. Acad. Sci. URSS, 563–568, 595–600.
- [4] Mursaleen M. A., Kilicman A. and Nasiruzzaman Md., (2022), Approximation by *q*-Bernstein-Stancu-Kantorovich operators with shifted knots of real parameters, *Filomat*, *36*(*4*):1179-1194.
- [5] Stancu D.D., (1983), Approximation of functions by means of a new generalized Bernstein operator, Calcolo, 20: 211–229. doi: 10.1007/BF02575593

### Lightlike Hypersurfaces of Meta-Metallic Semi-Riemannian Manifolds

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key-words: Metallic semi-Riemannian manifold, meta-metallic-Chi ratio, meta-metallic structure, lightlike hypersurface.

#### Abstract:

In this paper, we introduce lightlike hypersurfaces of a new developed manifold called meta-metallic semi-Riemannian manifold by using meta-metallic-Chi ratio. We obtain properties of induced structures on a lightlike hypersurface by terms of the meta-metallic semi-Riemannian structure. Especially, we define some subclasses of lightlike hypersurfaces namely invariant, anti-invariant and screen semi-invariant lightlike hypersurfaces of meta-metallic semi-Riemannian manifolds.

- [1] Barlett C., (2019), Nautilus spirals and the Meta-Golden ratio Chi, exus Netw J., 21, 641-656
- [2] Şahin F., Şahin B., (2021), Meta-golden Riemannian manifolds, *Mathematical Methods in the Applied Sciences*, 45(16), 8965-10501
- [3] De Spinadel V.W., (1999), The metallic means family and multifractal spectra, *Nonlinear Analysis Series B: Real World Applications*, 36(6), 721-745
- [4] Acet B.E., Lightlike hypersurfaces of metallic semi-Riemannian manifolds, *International Journal of Geometric Methods in Modern Physics*, 15(12), 16
- [5] Hretcanu C.E., Blaga A.M., (2018), Meta-golden Riemannian manifolds, *Differential Geometry-Dynamical Systems*, 20, 83-97
- [6] Erdoğan F.E., Yüksel Perktaş S., Bozdağ Ş.N., (2023), Meta-Metallic Riemannian manifolds, preprint

### A Study on Hypersurfaces of Meta-Metallic Riemannian Manifolds

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key-words: Metallic Riemannian manifold, meta-metallic-Chi ratio, meta-metallic structure, invariant hypersurface, non-invariant hypersurface.

#### Abstract:

In the present paper, we initiate the study of submanifolds in meta-metallic Riemannian manifolds by examining hypersurfaces. We present fundamental properties of induced structures on hypersurfaces provided by the meta-metallic Riemannian structure on the ambient manifold. We also obtain some characterizations for invariant and non-invariant hypersurfaces in such type of manifolds.

- [1] Hretcanu C.E., Crasmareanu M., (2013), Metallic structures on Riemannian manifolds, *Revista de la Union Matematica Argentina*, vol. 54, no. 2, pp. 15-27.
- [2] Blaga A.M., Hretcanu C.E., (2018), Invariant, anti-invariant and slant submanifolds of a metallic Riemannian manifold, *Novi* Sad Journal of Mathematics, vol. 48, no. 2, pp. 57-82.
- [3] De Spinadel V.W., (1999), The metallic means family and multifractal spectra, *Nonlinear Analysis Series B: Real World Applications, vol. 36, no. 6, pp. 721-745.*
- [4] Şahin F., Şahin B., (2021), Meta-golden Riemannian manifolds, *Mathematical Methods in the Applied Sciences, vol. 45, no.* 16, pp. 8965-10501.
- [5] Hretcanu C.E., Blaga A.M., (2018), Submanifolds in metallic Riemannian manifolds, *Differential Geometry-Dynamical Systems*, vol. 20, pp. 83-97.
- [6] Erdoğan F.E., Yüksel Perktaş S., Bozdağ Ş.N., (2023), Meta-Metallic Riemannian manifolds, preprint.

### **On Bernstein Schurer Type Operators Preserving Exponential Function**

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key-words: Bernstein-Schurer type operators, exponential function, Korovkin type theorem.

#### Abstract:

This paper is mainly focused on Bernstein-Schurer type operators which preserve exponential function. The structure of the operator that preserves the Korovkin-type test functions will be examined. The behavior of our operators will be analyzed by examining the result of uniform convergence, as well.

- [1] Aral, A., Cardenas-Morales, D. and Garrancho, P., 2018, Bernstein-type operators that reproduce exponential functions, *J. Math. Inequal, vol. 12, pp. 861-872.*
- [2] Ozsarac, F. and Acar, T., 2019, Reconstruction of Baskakov operators preserving some exponential functions, *Mathematical Methods in the Applied Sciences, vol.42, pp. 5124-5132.*
- [3] Yilmaz, O., Bodur, M. and Aral, A., 2018, On approximation properties of Baskakov-Schurer-Szasz operators preserving exponential functions, *Filomat*, vol.32, pp.5433-5440.

## **Independence Test for Family of Archimedean Copulas**

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key-words: Anderson-Darling statistic, Archimedean copula, Bernstein polynomial, Cramér–von Mises statistic, goodness of fit tests, Kendall distribution function, Kolmogorov-Smirnov statistic.

#### Abstract:

In this study, we proposed two two independence tests for the members of Archimedean copulas family. The proposed tests are developed by the well-known statistics Anderson-Darling and Kolmogorov-Smirnov based on the empirical Kendall distribution function (and the Bernstein polynomial) and the Kendall distribution function. A Monte Carlo simulation study is conducted to establish the test critical values table and to observe the power performance of the current and proposed tests. A numerical example is also presented.

- [1] Susam SO, Ucer BH. Testing independence for Archimedean copula based on Bernstein estimate of Kendall distribution function. J Stat Comput Simul. 2018;88:2589–2599
- [2] Karakaş, A. M., and Doğan, M. (2017). Archimedean Copula Parameter Estimation with Kendall Distribution Function. *Journal of the Institute of Science and Technology*, 7(3), 187-198.

### **Some Fixed Point Results in** *R***-Modular** *b***-Metric-like Space**

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key-words: Modular *b*-metric, binary relation, metric like space

#### Abstract:

This study aims to achieve a new notation referred to as  $\mathscr{R}$ -modular *b*-metric-like space which is an analysis of modular metric-like space via the concept of *b*-metric. Subsequently, we prove some related fixed point theorems in the context of this notion.

- Khalehoghli S., Rahimi H. and Gordji M. E., (2020), Fixed point theorem in *R*-metric spaces with applications, *AIMS Mathematics*, 5, 4, 3125-3137.
- [2] Baghani, H., Ramezani M., (2017): A fixed point theorem for a new class of set-valued mappings in \(\mathcal{R}-\)complete (not necessarily complete) metric spaces, *Filomat*, 31, 12, 3875–3884.
- [3] Ege, M. E., Alaca, C., (2018): Some results for modular metric spaces and an application to system of linear equations, *Azerbaijan J. Math.*, 8(1), 3-14.
- [4] Al-Sulami, H. H., Ahmad, J., Hussain, N. and Latif, A., (2018): Relation theoretic contraction results with applications to nonlinear matrix equations, *Symmetry*, *10*, 767.
- [5] Chistyakov, V. V., (2010): Modular metric spaces, I: Basic concepts, Nonlinear Anal., 72, 1-14.
- [6] Sharma, Y., Jain, S., (2022): Fixed point results of  $\beta_{\gamma}$ -Geraghty type contractive mapping in modular metric-like spaces, Journal of Analysis, 30, 3, 1305–1321.
## **Dual Magnetic Curves and Flux Ruled Surfaces**

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key-words: Dual space, magnetic curve, flux ruled surface, killing vector field, lorentz force.

#### Abstract:

Initially, an investigation is conducted on the dual trajectories traced by the Frenet vectors of a dual curve in dual space through the dual Lorentz force. The present study identifies the dual flux ruled surfaces that correspond to the Killing vectors on the dual unit sphere. Additionally, the study provides certain characterizations pertaining to these surfaces.

- [1] Barros M., Cabrerizo J.L., Fernández M. and Romero A., (2007), Magnetic vortex filament flows, *Journal of Mathematical Physics*, 48(8), 1-27
- [2] Druţá-Romaniuc S. L. and Munteanu, M. I., (2013), Killing magnetic curves in a Minkowski 3-space. Nonlinear Analysis: Real World Applications, 14(1), 383-396
- [3] Bozkurt Z., Gök I., Yayli Y. and Ekmekci F. N., (2014), A new approach for magnetic curves in 3D Riemannian manifolds, Journal of Mathematical Physics, 55(5), 1-12
- [4] Inoguchi J. I. and Munteanu M. I., (2022), Slant curves and magnetic curves, *Contact geometry of slant submanifolds*, 199-259
- [5] Bayram E., Guler F. and Kasap E., (2023), Magnetic flux ruled surfaces, *Mathematical Methods in the Applied Sciences*, 46(5), 5989-6001

## Nonlinear Fractional Boundary Value Peoblems on An Infinite Interval

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key-words: fractional differential equations, Riemann-Liouville fractional derivative, fixed point theorems, fractional integral boundary conditions.

#### Abstract:

Fixed point theory is of great importance in the field of mathematics because it has wide application areas such as nonlinear equation systems, integral equations, and economic models. On the other hand, fractional differential equations have attracted the attention of researchers because they are modelling of complex natural phenomena. Application areas such as control theory, mathematical biology, blood flow problems, earthquake vibration movements, movements of fluids are modeled by fractional calculus.[1, 2] Considering these, in this talk we study the existence results of positive solutions of the nonlinear Riemann-Liouville fractional boundary value problem

$$\begin{cases} D_{0+}^{\alpha}u(t) + p(t)f(t,u(t)) = 0, & 0 < t < \infty, \quad 2 < \alpha \le 3, \\ u(0) = u'(0) = 0, & D^{\alpha-1}u(\infty) = \sum_{i=1}^{m-2} \int_{0}^{a_{i}} a_{i}(t)D^{\beta_{i}}u(t)dt + \beta I^{q}u(\eta), \end{cases}$$
(3)

where  $D^{\alpha}$  and  $D^{\beta_i}$  are Riemann-Liouville fractional derivatives,  $\eta \in (0, \infty)$ ,  $0 < \beta_i < \alpha - 1$ ,  $\beta > 0$ ,  $a_i \in C([0, \infty), [0, \infty))$ ,  $f \in C([0, \infty) \times [0, \infty), [0, \infty))$  and  $I^q$  is Riemann-Liouville fractional integral,  $p : [0, \infty) \to [0, \infty)$  is not identical zero on any closed subinterval of  $[0, \infty)$  with  $\int_0^{\infty} p(s) ds < \infty$ . Here, the Guo-Krasnoselksii fixed point theorem on cone and the Leggett Williams fixed point theorem are used in order to obtain the existence results of positive solutions of fractional boundary value problem (1).

- [1] Podlubny, I., 1999, Fractional Differential Equations, Academic Press, San Diego.
- [2] Miller, K. S. and Ross, B., 1993, An Introduction to the Fractional Calculus and Differential Equations, John Wiley, New York.

## **Geometry of Dual Umbrella Matrices**

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key-words: Dual Matrices, umbrella matrices, screw motion

#### **Abstract:**

Umbrella matrices and their geometry have been studied in works [1, 2, 3], and the generalized form of [3] is addressed in [4]. In this study, for the first time, we obtained Dual Umbrella matrices using the Cayley Formula for Dual antisymmetric matrices with zero row sums. We provided the relationship between the antisymmetric matrix and the Darboux matrix of the motion. We demonstrated that Dual Umbrella matrices represent a screw motion in 3-dimensional Euclidean space and obtained the rotation angle and translation amount associated with this motion. Furthermore, we showed that these matrices are Circulant matrices.

- [1] Ozdamar E., (1977): The lie group of umbrella matrices and differential geometry, *Ph.D.Dissertation*, University of Ankara, Turkey
- [2] Esin E., (1986), Umbrella matrices and higher curvatures of a curve, *Commun. Fac. of Sci. Uni. of Ankara Seri. A1 Math. and Stat.*, 35, 28–34
- [3] Kuruoglu N., (1983), On the lie group of umbrella matrices, *Commun. Fac. of Sci. Uni. of Ankara Seri. A1 Math. and Stat.*, 32, 132–144
- [4] Carboga M., and Yayli Y., (2023), Geometric Applications and Kinematics of Umbrella Matrices, submitted
- [5] Mortazaasl H., Jafari M. and Yayli Y., (2011), Homothetic Motions in the Dual 3-Spaces, Int. J. Contemp. Math. Sciences, 6, 841-852
- [6] Yüca G., (2021), Kinematics applications of dual transformations, J. Geo. and Phys., 163, 104139
- [7] Bottema O. and Roth B., (1990): Theoretical kinematics, 24, Courier Corporation, ABD
- [8] Courant R. and Hilbert D., (2008): *Methods of mathematical physics: partial differential equations*, 2, John Wiley and Sons, Singapore.

## **Approximation by Complex Modified Stancu Operators**

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key-words: Perturbed Bernstein operator, complex Stancu operator, quantitative estimates, simultaneous approximation, equivalence.

#### Abstract:

In this talk, we introduce a new generalization of complex Stancu operators and study some approximation properties of these operators attached to analytic functions in an open disk centered at the origin. At first, we obtain upper quantitative estimates for the convergence and then give lower estimates from a qualitative Voronovskaja type theorem. Finally, we establish the exact degree of simultaneous approximation with the help of upper and lower estimates.

- [1] Acu, A. M. and Başcanbaz-Tunca, G., 2020, Approximation by complex perturbed Bernstein-type operators, *Results Math.*, 75(3):120.
- [2] Çetin, N. and Başcanbaz-Tunca, G., 2019, Approximation by a new complex generalized Bernstein operator, *An. Univ. Oradea Fasc. Mat., Tom XXVI; 2: 127–139.*
- [3] Gal, S. G., 2009, Approximation by Complex Bernstein and Convolution Type Operators. Series on Concrete and Applicable Mathematics, 8. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, USA.
- [4] Stancu, D. D., 1982, Quadrature formulas constructed by using certain linear positive operators, *Numerical Integration*, *ISNM 57*, 241-251, *Birkha "user Verlag, Basel.*
- [5] Stancu, D. D., 1983, Approximation of functions by means of a new generalized Bernstein operator, Calcolo, 20: 31 211–229. doi: 10.1007/BF02575593.

# Inference for Overlapping Coefficient of Two Bathtub-Shaped Lifetime Distributions Under k-Record Values

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key-words: Bathtub-shaped lifetime distribution, overlap measure, record values

#### Abstract:

This study deals with the inference of the overlapping measure for two bathtub-shaped lifetime distributions defined by Chen [1]. Among different overlapping coefficients, we use the most popular measure, Matusita's measure [2], since its simple mathematical structure and some strong properties such as consistency, unbiasedness, validity and etc. We consider the overlapping of these two populations under upper k- record values introduced by Dziubdziela and Kopociński [3] as the generalization of classical upper-record values. The maximum likelihood and Bayesian inference methods are used as frequentist and alternative methods, respectively. Point estimations of overlapping coefficient and its corresponding approximate confidence intervals are obtained by using both methods. The study ends with a numerical example to illustrate the theoretical outcomes.

- [1] Chen Z., (2000): A new two-parameter lifetime distribution with bathtub shape or increasing failure rate function. *Statistics & Probability Letters*, 49(2), 155-161.
- [2] Matusita K., (1955): Decision rules, based on the distance, for problems of fit, two samples, and estimation. *The Annals of Mathematical Statistics*, 631-640.
- [3] Dziubdziela W., Kopociński B., (1976): Limiting properties of the k-th record values. *Applicationes Mathematicae*, 2(15), 187-190.

## On the Algebra of Elliptical Dual Quaternions and Hamilton Matrices

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key-words: Elliptic Quaternion, Elliptic Dual Quaternion Algebra, Hamilton Operators

#### Abstract:

Quaternions were described by the Irish mathematician W.R. Hamilton in 1843. By defining a new multiplication on the set of quaternions, Hamilton has facilitated the investigation of motions in Euclidean space. In this study, algebraic structure of elliptic dual quaternions has been constructed and some properties of these kind of quaternions have been investigated with the help of Hamilton matrices.

- [1] Vinberg, E.B., (2003), A Course in Algebra, Graduate Studies in Mathematics, *American Mathematical Society, Providence, Rhade Island.*
- [2] Gallian, J.A., (2013): Contemporary Abstract Algebra, Brooks Cole.
- [3] Hacisalihoğlu, H.H., (1983), Motion Geometry and Quaternions Theory, *Gazi University, Faculty of Arts and Sciences Publications Mathematics*, 2.
- [4] Özdemir, M., (2020), Quaternions and Geometry, Golden Point Publishing.

## Generalized Geometric Phase Applications in the Optic Fiber with the Kinematics of Framed Curve

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key-words: Applications to physics, magnetic flows, vector fields, ordinary differential equations, electromagnetic theory, framed curves.

#### Abstract:

In this study, we have developed the Berry phase models for the framed curve in the optical fiber. Our analysis relies on the definition of a moving attached to a framed curve, and the polarization state defined at every point on the curve and described in terms of the moving frame. This formulation provides a description of polarization state for curves that may have singularities. Then, by means of the obtained Berry phase models, we generate the electromagnetic curves. Moreover, these forms of polarization states give us the polarization vector parallel transported along the optical fiber with the Fermi Walker parallel transportation rule. Several examples are considered to demonstrate the computational behaviour of the polarization state. The assumption that the framed curve has singularities gives Rytov curves that have singular points. Also the curvature of the framed curve is quite useful to analyse the framed curves and their singularities. The description we have provided here is a geometric approach to the polarization state in the optical fiber, but from our analysis we can infer several characteristics of the underlying physics. To elaborate, we consider the electromagnetic curves. The charged-point particle motions in the optical fiber are generated by the Lorentz force related to the electric field and magnetic field called the electromagnetic curves. Our analysis enables us to deduce how the behave of the charged particle motion along the framed curve that may have singular points. We found in the third section also that the polarization state and the related electromagnetic curves when the constant angle between the polarization vector and the moving frame related to the framed curve in the optical fiber. This gives that the polarization vector not only rotate it also translation in the optical fiber, (see [6]). The geometrical analysis of Berry phase models in the optical fiber construction can provide useful insight into electromagnetic theory to connect the geometric description directly to physics.

- [1] Barros M., (1997), Magnetic helices and a theorem of Lancret, Proc. Amer. Math. Soc., 125(5), 1503-1509
- [2] Bozkurt Z., Gök İ., Yaylı Y. and Ekmekci F. N., (2014): A new approach for magnetic curves in 3D Riemannian manifolds J. Math. Phys., 55, 053501
- [3] Bruce J. W., Giblin P. J., (1992), Curves and singularities, Cambridge Univ. Press
- [4] Honda S., Takahashi M., (2016), Framed curves in the Euclidean space, Adv. Geom., 16(3), 265-276
- [5] Keskin Ö., Yaylı Y., (2017), Normal Fermi-Walker Derivative, Math. Sci. and Appl. E-Notes., 5(1), 1-8
- [6] Özdenir Z., (2020), A New Calculus for the Treatment of Rytov's Law in the Optical Fiber, *Optik International Journal for Light and Electron Optics.*, 216, 164892

## **Fuzzy Initial Value Problem with Dirac Delta Function**

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key-words: Dirac delta function, fuzzy initial value problem, fuzzy number.

### Abstract:

In this study the fuzzy differential equation containing the dirac-delta function is considered. Based on this notion, the fuzzy solutions are obtained for fuzzy initial value problems by means of the fuzzy laplace transform. Finally, two examples are given in order to compare the proposed solutions.

## References

[1] Bede B., Stefanini L., (2013), Generalized differentiability of fuzzy-valued functions, Fuzzy Sets and Syst., 230, 119-141

- [2] Friedman M., Ma M., Kandel A., (1999), Numerical solution of fuzz differential and integral equations, *Fuzzy Sets and Syst.*, 106(35–48), 35-48
- [3] Khastan A., Bahrami F., Ivaz K., (2009), New Results on Multiple Solutions for Nth-Order Fuzzy Differential Equations under Generalized Differentiability, *Boundary Value Problems*, 70, 1-13
- [4] Salahshour S., Allahviranloo T., (2009), Applications of fuzzy Laplace transforms, Soft Computing, 17(1), 145-158
- [5] Dirac P., (1958): The principles of Quantum Mechanics. 4th edn., World Scientific

## Combining Chaotic Dynamical Systems Using the Fuzzy Logic XOR Operator

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key-words: Chaos, fuzzy logic, ergodic theory, full branch.

#### Abstract:

Chaotic dynamical systems are commonly used as models in a wide range of applications, cryptography [1, 5, 9, 11] image and speech encryption and retrieval [2, 6, 8] and achievement of associative memory properties [3, 4].

Extreme sensitivity to initial conditions is an interesting property of chaotic systems. This property makes chaotic systems a worthy choice for constructing cryptosystems or for image encryption.

Another idea is to mix two or more chaotic dynamical systems to gain more "unpredictably" in order to enhance encryption process [6, 7].

We ask ourselves this question, whatever combining two chaotic dynamical systems permits to maintain chaotic property of the resulting one ? In particular whatever combining two chaotic dynamical systems by fuzzy logic operators, mainly the xor operator gives rise to a chaotic dynamical system.

In other words consider two chaotic dynamical systems (I, F) and (I, G) is the dynamical system (I, F x or G) still chaotic?

We studied some combination of known chaotic dynamical systems and checked whatever the combination using xor operator is still chaotic or not. This gave us some preliminary remarks about how to combine chaotic dynamical systems in order to maintain the chaotic properties of the resulting dynamical system.

We provide a sufficient condition to check if the combination has chances to succeed. Rather than the popular Lyapunov exponents method used almost systematically in the applied mathematics literature we use a tool provided by ergodic theory [10].

- [1] Baptista M. S. Cryptography with chaos / Baptista M. S. Physics Letters A. 240(1-2). 1998. P. 50-54.
- [2] R. Chemlal, I Djellit, Coding Information and Problems of storage in Dynamical Systems, Facta Universitatis, Ser Elec Energ, Vol 17, December 2004, 355-363.
- [3] A.A. Dmitriev Design of Message-Carrying Chaotic Sequences 2002 Nonlinear Phenomena in Complex Systems.
- [4] H. K. Kwan, Three Layer Bi directional asymmetrical associative memory IEEE 2003.
- [5] Kocarev L. Chaos-based cryptography: A brief overview, IEEE Circuits and Systems Magazine. 2001. N 1. P.6-21.
- [6] Mykola Kushnir, Yuriy Fedkovych, Petro Kroialo, Hryhorii Kosovan. Encryption of the Images on the Basis of Two Chaotic Systems with the Use of Fuzzy Logic. 2020 IEEE 15th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET).
- [7] N.K. Pareek a,b, Vinod Patidar. Cryptography using multiple one-dimensional chaotic maps, K.K. Sud. Communications in Nonlinear Science and Numerical Simulation 10 (2005) 715–723
- [8] S.Rouabhi "Storage of Information in One Dimensional Piecewise Continuous Maps" International Journal of Bifurcation and Chaos. (2000) vol 10(5) pp 1127-1137.
- [9] Schwarz W. D. Chaos and cryptography / Schwarz W. D., Dachselt W. IEEE Trans. Circuits Syst. I. 48 (12). 2001.1498-1509.
- [10] S.Luzzato, Introduction to smooth ergodic theory, available online at https://indico.ictp.it/event/a12289/session/2/contribution/1/material/0/0.pdf
- [11] P.G. Vaidya, S. Angadi, Decoding chaotic cryptography without access to the superkey, , Chaos, Solitons and Fractals 17 (2003) 379-386.
- [12] M. Vellekoop and R. Berglund. On intervals, transitivity = chaos. Amer. Math. Monthly, 101(4):353-355, 1994.

## **Two Variable Higher-order Generalized Fubini Polynomials**

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key-words: Generating function, multilinear and multilateral generating functions, recurrence relation, integral representation.

#### Abstract:

This article attempts to present two variable higher-order generalized Fubini polynomials  $F_{n,\lambda}^{(r)}(x,y)$ . The results obtained here include various families of multilinear and multilateral generating functions, various properties, as well as some special cases for two variable higher-order generalized Fubini polynomials  $F_{n,\lambda}^{(r)}(x,y)$ . Finally, we get several interesting results of this two variable higher-order generalized Fubini polynomials and obtain an integral representation.

## References

[1] Kargin, L.; Some formulae for products of Fubini polynomials with applications. arXiv 2016, arXiv:1701.01023v1

- [2] Ozmen, N. and Erkus-Duman, E.; Some results for a family of multivariable polynomials, AIP Conf. Proc. 1558,(2013), 1124– 1127
- [3] Kilar, N., Simsek, Y.; A new family of Fubini type numbers and polynomials associated with Apostol-Bernoulli numbers and polynomials. J. Korean Math. Soc. 2017, 54, 1605–1621.
- [4] Su, D.D., He, Y.; Some identities for the two variable Fubini polynomials. Mathematics 2019, 7–115.
- [5] Ozmen, N.; Bilateral and Bilinear generating functions for the modified generalized Sylvester Polynomials, Facta Universitatis (NIS), 33(2), (2018), 279–293.
- [6] Korkmaz-Duzgun, D., Erkus-Duman, E.; Generating functions for k-hypergeometric functions. Int. J. Appl. Phys. Math. 2019, 9, 119–126.
- [7] Masjed-Jamei, M., Beyki, M. R. and Koepf, W.; A new type of Euler polynomials and numbers, Mediterr. J. Math. 15(2018), 3, 138.
- [8] Kim, D.S., Kim, T., Kwon, H.-I., Park, J.-W.; Two variable higher-order Fubini polynomials. J. Korean Math. Soc. 2018, 55, 975–986.
- [9] Ağca, M. and Ozmen, N.; Miscellaneous Properties of Generalized Fubini Polynomials. Commun. Adv. Math. Sci. 2023, 6(1), 19-30.

## **Approximation via Power Series Statistical Convergence of Matrix-valued Functions**

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key-words: Matrix-valued functions,  $P_p^2$ -statistical convergence, Korovkin theorem, rate of approximation.

#### **Abstract:**

In this study, we give an extension of Korovkin's theorem for functions belonging to a special class. Basically, we will obtain effective results by making use of the notion of statistical convergence in sense of the power series method which is widely used in approximation theory. With given example, we offer a stronger point of view than the classical meaning and we study the the rates of  $P_P^2$ -statistical convergence.

- Baron S., (1997), Tauberian theorems for power series methods applied to double sequences. J. Math. Anal. Appl. 211(2),574-589
- [2] Duman O. ve Erkuş-Duman E., (2011), Statistical Korovkin-type theory for matrix-valued functions. Studia Scientiarum Mathematicarum Hungarica, 48(4), 489-508
- [3] Korovkin P.P., (1960), Linear Operators and Approximation Theory, Hindustan Publ. Co., Delhi
- [4] Moricz F., (2004), Statistical convergence of multiple sequences, Arch. Math. (Basel), 81, 82-89
- [5] Serra-Capizzano S., (1999), A Korovkin based approximation of multilevel Toeplitz matrices (with rectangular unstructured blocks) via multilevel trigonometric matrix spaces, SIAM J. Numer. Anal., 36(6), 1831-1857
- [6] Yıldız S., Demirci K., and Dirik F., (2023) Korovkin theory via Pp-statistical relative modular convergence for double sequences. Rendiconti del Circolo Matematico di Palermo Series 2, 1-17

## **Gaussian Generalized Ernst Numbers**

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key-words: Gaussian Ernst numbers, Gaussian Ernst-Lucas numbers, Gaussian generalized Ernst numbers.

#### Abstract:

In the present paper, we investigate the Gaussian generalized Ernst numbers and also we exhibit generating functions, Binet's formulas, Simson formulas for Gaussian Ernst and Gaussian Ernst-Lucas numbers. Additionally, we construct some identities and matrices related to these sequences. The most important detail that the reader will find in this study is that the Generalized Ernst numbers are defined for the first time in the Gaussian sense.

- [1] Soykan, Y., (2022), Generalized Ernst Numbers, Asian Journal of Pure and Applied Mathematics, vol. 4, pp. 136-150.
- [2] Pethe S. and Horadam A.F., (1986), Vol.33, pp.37-48, Generalised Gaussian Fibonacci numbers, Bull. Austral. Math. Soc.vol.33, pp.37-48.
- [3] Berzsenyi G., (1977), Gaussian Fibonacci Numbers, Fibonacci Quarterly, vol.15, pp.233-236.

## Generalized Split Quaternions and Their Applications on Non-parabolic Conical Rotations

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key-words: Lorentzian inner product, Lorentzian vector product, Lorentzian rotation matrix, split quaternion.

#### Abstract:

The aim of the study is giving a generalization for split quaternions and determining non-parabolic conical rotations using the generalized split quaternions. First we define the generalized Lorentzian scalar product whose sphere is any given hyperboloid one or two sheets having equation

 $Ax^{2} + By^{2} + Cz^{2} + 2Dxy + 2Exz + 2Fyz = \pm r^{2}$ 

and then determine the generalized Lorentzian vector product. Finally, we define the generalized split quaternions using these products, and we determine the elliptic and hyperbolic rotational motions on hyperboloids using the generalized split quaternions, providing numerical examples.

- [1] Çolakoğlu H.B., Öztürk İ., Özdemir M., (2023) Non-parabolic conical rotations, *Journal of Computational and Applied Mathematics*, vol. 420, 114766, pp. 1-15.
- [2] Çolakoğlu H. B. and Özdemir M., (2023) Generalized elliptical quaternions with some applications, *Turkish Journal of Mathematics*, vol. 47, pp. 351-371.
- [3] Erdoğdu, M., Özdemir, M., (2015) On reflections and rotations in Minkowski 3-space of physical phenomena, *Journal of Geometry and Symmetry in Physics, vol. 39, pp. 1-16.*
- [4] Şimşek, H., Özdemir, M., (2016) Generating hyperbolical rotation matrix for a given hyperboloid, *Linear Algebra and its Applications, vol. 496, pp. 221-245.*

## Effectiveness of the Kashuri Fundo Decomposition Method in Solving Fractional Differential Equations: A Case Study on the Fornberg-Whitham Equation

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key-words: Kashuri Fundo decomposition method, fractional differential equation, Fornberg-Whitham equation.

#### Abstract:

Fractional differential equations (FDEs) have become a focal point in various scientific and engineering fields due to their ability to accurately model complex phenomena [1, 2]. Solving FDEs presents challenges due to the non-local and non-linear nature of fractional derivatives. In this study, we investigate the effectiveness of the Kashuri Fundo decomposition method [3] as a powerful numerical technique for solving FDEs, with a specific focus on the Fornberg-Whitham equation.

By employing the Kashuri Fundo decomposition method, the Fornberg-Whitham equation is transformed into an algebraic equation by introducing auxiliary functions. This method provides a practical and efficient approach to obtain numerical solutions for the Fornberg-Whitham equation.

The obtained results are compared with the results obtained by other methods in the literature. The results revealed with this method are in perfect harmony with some of the results revealed with the existing methods, while in some they are closer to the exact solution than others.

In conclusion, this study establishes the Kashuri Fundo decomposition method as an effective numerical technique for solving FDEs, demonstrated through its successful application to the Fornberg-Whitham equation.

Further research is encouraged to explore the method's potential in solving other complex FDEs in diverse scientific and engineering applications.

- [1] Miller K. S. and Ross B., (1993): An Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley-Interscience
- [2] Debnath L., (2003), Recent Applications of Fractional Calculus to Science and Engineering, International Journal of Mathematics and Mathematical Sciences, vol. 54, pp. 3413-3442
- [3] Sumiati I., Sukono and Bon A. T., (2020): Adomian Decomposition Method and The New Integral Transform, Proceedings of the 2nd African International Conference on Industrial Engineering and Operations Management, Harare, pp. 1882-1887

## Ensemble based Type-1 Fuzzy Functions Approach for Time Series Forecasting

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key-words: Time series forecasting, ensemble, fuzzy functions.

#### Abstract:

This study introduces an innovative approach for time series forecasting, leveraging type-1 fuzzy functions and ensemble learning. The methodology employs a matrix to capture the dynamic nature of time series data, which is then classified into clusters using fuzzy C-means (FCM) clustering algorithm. An ensemble model uses the FCM memberships to predict future values, thus enabling the capturing of intricate temporal dependencies and imprecise characteristics of real-world data. Initial results show that our model demonstrates a strong potential for accurate forecasting of complex time series. By merging the interpretability of type-1 fuzzy functions with the robust predictive power of ensemble learning, this work presents a significant advancement in time series forecasting methodologies.

## On a Generalization of the Operators Involving Apostol-Genocchi Polynomials

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key-words: Apostol-Genocchi polynomials, rate of convergence, Korovkin theorem, modulus of continuity, weighted space of functions.

#### Abstract:

In the present paper we deal with the operators involving Apostol-Genocchi polynomials. We propose a generalization of the mentioned operators and we focus on the approximation properties of the newly defined operators including, local approximation, rate of convergence in terms of first and second modulus of continuity and Peetre's K-functional. We also examine the convergence of the operators in a weighted space.

- [1] Ciupa, A., 1995, A class of integral Favard-Szász type operators, Stud. Univ. Babes-Bolyai, Math, vol. 40(1), pp. 39-47.
- [2] Deo, N. and Kumar, S., 2021, Durrmeyer variant of Apostol-Genocchi-Baskakov operators, *Quaest. Math, vol. 44(12), pp. 1817-1834.*
- [3] Luo, Q. M., (2009), q-Extensions for the Apostol-Genocchi polynomials, General Mathematics, vol. 17(2), pp. 113-125.
- [4] Luo, Q. M., 2011, Extensions of the Genocchi polynomials and their Fourier expansions and integral representations, *Osaka J. Math.*, vol. 48, pp. 291-309.
- [5] Neha and Deo N., 2021, Integral modification of Apostol-Genocchi operators, Filomat, vol. 35(8), pp. 2533-2544.
- [6] Neha and Deo N., 2023, Integral modification of Beta-Apostol-Genocchi operators, *Mathematical Foundations of Computing*, vol. 6(3), pp. 474-483.
- [7] Prakash, C., Verma, D. K. and Deo, N., 2021, Approximation by a new sequence of operators involving Apostol-Genocchi polynomials, *Mathematica Slovaca*, vol. 71 (5), pp. 1179-1188.
- [8] Menekşe, Yilmaz M., 2022, Approximation by Szasz Type Operators Involving Apostol-Genocchi Polynomials, CMES-Computer Modelling in Engineering and Science, vol. 130 (1), pp. 287-297.

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### **Almost Borderenergetic Line Graphs**

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key-words: Borderenergetic graph, almost borderenergetic graph, line graph.

### Abstract:

Let G be such a graph of order n, and  $\lambda_1 \ge \lambda_2 \ge ... \ge \lambda_n$  be its eigenvalues, i.e., the eigenvalues of the adjacency matrix A(G) of G. These eigenvalues form the *spectrum* of the graph G, which will be denoted by Sp(G); for details on spectral graph theory see [5]. The energy of the graph G is the sum of the absolute values of the eigenvalues of A(G), denoted by E(G), is defined as [10]

$$E\left(G\right) = \sum_{i=1}^{n} \left|\lambda_{i}\right|.$$

Recently, Gong et al. [8] proposed the concept of borderenergetic graphs, a graph G of order n is said to be *borderenergetic* if its energy equals the energy of the complete graph  $K_n$ , i.e., if E(G) = 2(n-1). On the other hand, graphs of order n having energy close to 2(n-1) is also interesting. We define a graph of order n with |E(G) - 2(n-1)| < 1 almost borderenergetic graph. Let G be a graph with edge set E. The line graph of G will be denoted by  $\mathcal{L}(G)$ , is the graph whose vertex set is E; two vertices of  $\mathcal{L}(G)$  are adjacent if and only if the corresponding edges in G are adjacent. The basic properties of line graphs are found in text books, such as [14]. We also note that the join  $G = G_1 \nabla G_2$  of graphs  $G_1$  and  $G_2$  is defined as  $G = \overline{G_1 \cup G_2}$ , where  $\cup$  represents the union operator on two graphs and  $\overline{G}$  is the complement of the graph G. Similarly, the union of r-copies of G is shown as  $rG = \underline{G \cup G \cup \cdots \cup G}$ .

In this study, we firstly present the spectrum of graph  $\mathcal{L}(rK_1\nabla K_2)$  and show that  $\{\mathcal{L}((rK_1)\nabla K_2)|r \geq 5\}$  is an infinite family of connected almost borderenergetic graph. In particular, we see that energy of  $\mathcal{L}((rK_1)\nabla K_2)$  reaches to 4r as r increases, which defines an family of *asymptotic borderenergetic graphs*.

### References

r-copies

- [1] Cvetković D., Rowlinson P. and Simić S., (2009), An Introduction to the Theory of Graph Spectra, Cambridge Univ. Press, Cambridge
- [2] Gong S. C., Li X., Xu G. H., Gutman I. and Furtula B., (2015), Borderenergetic graphs, *MATCH Commun. Math. Comput. Chem.*, 74, 321-332
- [3] Gutman I., (1978), The energy of a graph, Ber. Math. Statist. Sekt. Forschungsz. Graz., 103, 1-22
- [4] Harary F., (1969), Graph Theory (Chapter 8), Addison-Wesley, Reading

## On The Existence of MDS Matrices over $\mathbb{F}_p + v\mathbb{F}_p$

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key-words: MDS matrices, circulant, orthogonal.

#### Abstract:

An  $n \times n$  matrix is called MDS (Maximum Distance Separable) matrix if and only if its submatrices are non-singular. In 2022, Adhiguna et al proved that over a field of characteristic p > 2 there is no orthogonal circulant MDS matrix of even order m and of order divisible by p. In this research, we observe the existence of MDS matrices over ring  $\mathbb{F}_p + v\mathbb{F}_p$  where  $v^2 = v$ . Using the fact that for every  $a + vb \in \mathbb{F}_p + v\mathbb{F}_p$  can be written as  $a + vb = v(a + b) \oplus (1 - v)a$ , we prove that there is no orthogonal circulant MDS matrix of even order and of order divisible by p over  $\mathbb{F}_p + v\mathbb{F}_p$ .

- Adhiguna I., Arifin I.S.N, Yuliawan F., Alamsyah M.I., (2022), On Orthogonal Circulant MDS Matrices, International Journal of Mathematics and Computer Science, 17, no. 4, 1619-1637
- [2] Daemen J., Rijmen V., (2020): The design of Rijndael: the Advanced Encryption Standard (AES), Springer
- [3] Cauchois, V., Loidreau, P. (2019), On circulant involutory MDS matrices, Des. Codes Cryptogr. 87, 249-260
- [4] Irwansyah, Barra S., Muchtadi-Alamsyah I., Muchlis A., and Suprijanto D., (2017), Skew-cyclic codes over B<sub>k</sub>, Journal of Applied Mathematics and Computing (to appear) DOI 10.1007/s1219001710952
- [5] Gupta, K. C. and Ray, I. G. (2015), Cryptographically significant MDS matrices based on circulant and circulant-like matrices for lightweight applications, *Cryptography and Communications*, 7, 257-287
- [6] MacWilliams F. J., Sloane N. J. A. (1997): The Theory of Error Correcting Codes, North-Holland Publishing Co., Amsterdam-New York-Oxford, 9-321.

# **Unbounded Convergence in** $C_p(X, [0, 1])$

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key-words:  $C_p(X, [0, 1])$ , Tychonoff space, unbounded convergence.

### Abstract:

 $C_p(X, [0, 1])$  denotes the space of all continuous [0,1] valued functions on a Tychonoff Space. In this talk, I will give some characterizations of unbounded convergent nets or sequences in space  $C_p(X, [0, 1])$ . Also, it will be shown that a sequence unbounded order converges if it converges pointwise on a co-meagre set.

- [1] Bilokopytov, E., Troitsky, V. G., (2022), Order and uo-convergence in spaces of continuous functions, *Topology and its* Applications, vol.308, 107999.
- [2] Osipov, A. V., (2021), Baire property of space of Baire-one functions, arXiv:2110.15496.
- [3] Osipov, A. V., Pytkeev, E. G., (2022), Baire property of spaces of [0,1]-valued continuous functions, *Revista de la Real* Academia de Ciencias Exactas, 117:38.

## On Nilpotent Leibniz Algebras with One Dimensional Leib Ideal

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key-words: Classification, nilpotency, congruence.

### Abstract:

Leibniz algebras are nonanticommutative generalization of Lie algebras. Classifying any kind of algebras is an interesting problem. It is known that classification of all nilpotent Lie algebras is still unsolved and wild problem. On the other hand, classifying nilpotent Leibniz algebras is more troublesome due to lack of anticommutative property in Leibniz algebras. The classification of low dimensional complex nilpotent Leibniz algebras with one dimensional Lieb ideal is given. We use the canonical forms for the congruence classes of matrices of bilinear forms given in [3] to classify these subclasses.

- Casas J.-M., Insua M.-A., Ladra M., Ladra S., (2012), An algorithm for the classification of 3-dimensional complex Leibniz algebras, *Linear Algebra Appl.*, 9, 3747-3756
- [2] Demir, I., (2018): Classification of 5-dimensional complex nilpotent Leibniz algebras, Contemporary Mathematics: Representations of Lie Algebras, Quantum Groups and Related Topics, 95-120, Raleigh
- [3] Teran, F.D., (2016), Canonical forms for congruence of matrices and T-palindromic matrix pencils: a tribute to H. W. Turnbull and A. C. Aitken, *SeMA*, *73*, *7-16*

## Statistical Korovkin Type Approximation Theorems for Double Sequences of Monotone and Sublinear Operators for Bögel Continuous Functions

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key-words: Bögel continuous functions, monotone and sublinear operators, nonlinear Choquet integral.

#### Abstract:

In this presentation, we obtain approximation theorems of Korovkin type for double sequences of monotone and sublinear operators defined on the space of Bögel continuous functions by means of statistical convergence in the sense of almost everywhere, in measure. Also, we give examples that satisfy our theorems. Finally, we calculate the rate of convergence.

- [1] Dirik F., Duman O. and Demirci K., (2010), Approximation in Statistical Sense to B-Continuous Functions By Positive Linear Operators, *Studia Scientiarum Mathematicarum Hungarica, vol.* 47(3), pp. 289-298
- [2] Badea C., Badea I. and Gonska H. H., (1986), A test function and approximation by pseudo-polynomials, *Bull. Austral. Math.* Soc.34(1986), 53-64
- [3] Korovkin, P.P., (1953), On convergence of linear positive operators in the space of continuous functions. (Russian), *Doklady Akad. Nauk. SSSR. (NS), vol. 90, pp. 961-964*
- [4] Anastassiou, G.A., (2018), Approximations by sublinear operators, Acta Math. Univ. Comenian., vol. 87(2), pp. 237-250
- [5] Gal, S.G., Niculescu, C.P., (2023), Korovkin type theorems for weakly nonlinar and monotone operators, *Mediterr. J. Math.*, vol. 20, pp. 1-20
- [6] Gal, S.G., Iancu, I.T., (2023), Korovkin-Type Theorems for Statistically Convergent Sequences of Monotone and Sublinear Operators, Bull. Malays. Math. Sci. Soc., vol. 46, pp. 1-15

## Picture Fuzzy Soft Topological Spaces and An Application of Multicriteria Group Decision Making Problems

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key-words: Picture fuzzy soft set, picture fuzzy soft topology, decision making problems, AHP-Topsis.

#### Abstract:

Picture fuzzy soft sets were defined by Coung as a hybrid model for vagueness. In this paper we set a topology on picture fuzzy soft sets and obtain the basic properties of this topology. We developed the AHP-TOPSIS (analytical hierarchy process and technique fort he order preference by similarity to ideal solution) for Picture fuzzy soft sets and presented an application to help families with their school choice problem.

- [1] Cuong, B.C., (2014), Picture fuzzy sets., J. Comput. Sci. Cybern. 30, 409-420.
- [2] Hwang C. L. and Yoon K., (1981), *Multiple Attributes Decision Making Methods and Applications*, Springer, Berlin Heidelberg.
- [3] Molodtsov, D., (1999), Soft set theory-first results, Comput. Math. Appl., 19-31.
- [4] Saaty, T.L., (2008), Decision making with the analytic hierarchy process., Int. J. Serv. Sci. 1, 83-98.

## A Variational Method Based on the Discrete *q*-Hermite II Polynomials to Obtain Spectrum of a *q*-deformed Schrödinger Equation

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key-words: Discrete Schrödinger equation, q-harmonic oscillator, Rayleigh-Ritz variational method, discrete q-Hermite II polynomials.

#### Abstract:

In the present work, the spectrum of a q-deformed Schrödinger equation is obtained by using the Rayleigh-Ritz variational method in which the discrete q-Hermite II polynomials are taken as the basis. The eigenvalue problem is reduced to a matrix eigenvalue problem for which some recursive relations have been obtained for the evaluation of matrix elements. The energy spectrum of the q-harmonic oscillator has been investigated as the first application. After that q-versions of purely quartic and quartic oscillators are considered. Furthermore, a specific example for q-version of asymmetric double well potential (q-ADWP) is presented. For each potential, a few energy levels corresponding to various q-values are obtained. It is seen that, in the limiting case as  $q \rightarrow 1^-$ , the eigenvalues in the discrete problem approach to those in the continuous case. Also, the behavior depending on the size of truncation matrix is examined for different values of q.

- [1] Alıcı H. and Taşeli H., (2010), Pseudospectral methods for solving an equation of hypergeometric type with a perturbation, *J. Comput. Appl. Math.*, vol. 234, pp. 1140-1152.
- [2] Kac V. and Cheung P., (2002): Quantum Calculus, Berlin: Universitext, Springer-Verlag.
- [3] Koekoek R., Lesky P. A. and Swarttouw R. F., (2010): *Hypergeometric orthogonal polynomials and their q-analogues*, Springer Monographs in Mathematics, Berlin-Heidelberg: Springer-Verlag.
- [4] Koekoek R. and Swarttouw R.F., (1998): *The Askey-scheme of hypergeometric orthogonal polynomials and its q-analogue*, Reports of the Faculty of Technical Mathematics and Informatics.
- [5] Nikiforov A. F., Suslov S. K. and Uvarov V. B., (1991): Classical Orthogonal Polynomials of a Discrete Variable, Springer Ser. Comput. Phys., Springer-Verlag, Berlin.
- [6] Taşeli H. and Erseçen, M. B., (2003), The scaled Hermite-Weber basis still highly competitive, *Journal of Mathematical Chemistry, vol. 34, pp. 177-187.*
- [7] Turan M., Sevinik Adıgüzel R. and Doğan Çalışır A., (2021), Spectrum of the q-Schrödinger equation by means of the variational method based on the discrete q-Hermite I polynomials, *International Journal of Modern Physics A, vol. 36 (3)* Article Number: 2150020.

## Existence of Solutions for Second Order Impulsive Boundary Value Problems on Time Scales

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key-words: BVPs, cone, k-set contraction, expansive operator, sum of operators, existence of solutions.

#### Abstract:

In this paper, we study the second order boundary value problem for impulsive dynamic equations of the form:

 $\begin{array}{rcl}
-x^{\Delta\Delta}(t) &=& f(t, x(t)), & t \in J \setminus \{t_1, \dots, t_m\}, \\
x(t_k^+) - x(t_k^-) &=& I_k(x(t_k^-)), & k \in \{1, \dots, m\}, \\
x^{\Delta}(t_k^+) - x^{\Delta}(t_k^-) &=& L_k(x(t_k^-)), & k \in \{1, \dots, m\}, \\
x(0) = x(1) &=& 0.
\end{array} \tag{4}$ 

We give conditions under which the considered problem has at least one and at least two solutions. In our case, it is possible  $I_k, L_k, k \in \{1, \ldots, m\}$  to be unbounded unlike most of the existing literature. Therefore the results in this paper can be considered as complementary results to previous studies.

The arguments are based upon recent fixed point index theory in cones of Banach spaces for a k-set contraction perturbed by an expansive operator. An example is given to illustrate the obtained results.

- [1] Agarwal, R. P.; Bohner, M. Basic calculus on time scales and some of its applications. Results in Mathematics, 35(1),1999 3-22.
- [2] R.P. AGARWAL, M. MEEHAN AND D. O'REGAN Fixed Point Theory and Applications, Cambridge University Press, Vol. 141, (2001).
- [3] Bainov, D.; Simeonov, P. Impulsive differential equations: periodic solutions and applications, CRC Press, 1993.
- [4] Banas, J.; Goebel, K. Measures of Noncompactness in Banach Spaces,
- [5] Benchohra, M.; Ntouyas, S.; Ouahab, A. Existence results for second order boundary value problem of impulsive dynamic equations on time scales, Journal of Mathematical Analysis and Applications, Vol. 296(2004), No. 1, pp. 65–73.
- [6] Bohner M.; Peterson A. Dynamic equations on time scales: An introduction with applications, Springer Science & Business Media, 2001.
- [7] Djebali, S.; Mebarki, K. Fixed Point Index Theory for Perturbation of Expansive Mappings by k-set Contraction, Top. Meth. Nonli. Anal., Vol 54, No. 2 (2019), 613–640.
- [8] Drabek, P.; Milota, J. Methods in Nonlinear Analysis, Applications to Differential Equations, Birkhäuser, 2007.
- [9] J. DUGUNDJI AND A. GRANAS, Fixed point theory, Monographie Mathematycznz, Vol. 1, PNW Warsaw, 1982.
- [10] Georgiev, S; Kheloufi, A; Mebarki, K. Classical solutions for the Korteweg-De Vries equation, New trends Nonlinear Anal. Appli., (2022), (to appear).
- [11] Georgiev, S; Zennir, K. Existence of solutions for a class of nonlinear impulsive wave equations., Ricerche mat (2021), https://doi.org/10.1007/s11587-021-00649-2Lecture Notes in Pure and Applied Mathematics, 60. Marcel Dekker, Inc., New York, 1980.
- [12] Hilger, S. Ein maßkettenkalkül mit anwendung auf zentrumsmannigfaltigkeiten. PhD thesis, Universität Würzburg, 1988.
- [13] Lakshmikantham, V.; Simeonov, P. Theory of impulsive differential equations, World scientific, 6, 1989.
- [14] Li, J.; Shen, J. Existence results for second-order impulsive boundary value problems on time scales, Nonlinear Analysis: Theory, Methods & Applications, Vol. 70(2009), No. 4, pp. 1648-1655.
- [15] H. Schaefer, Uber dei, Methods der approiri, Schranken Math. Ann No. 129, (1955)pp. 415-416.
- [16] D.R. Smart. Fixed point theorems. Cambridge University Press, Cambridge, 1974.

## **On Outer Billiards**

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key-words: Dynamical systems, hyperbolic plane, billiards, outer billiards, periodic orbits.

### Abstract:

In this talk, we report on research about "the orbit structures of the outer (dual) billiard map in the hyperbolic plane". In particular, after giving a brief introduction to billiard type of dynamical systems and Hyperbolic geometry, we shall discuss the geometric properties of 3-periodic orbits of the outer billiard map in the hyperbolic plane.

- [1] Dogru F., Tabachnikov S., (2003), On polygonal dual billiards in the hyperbolic plane., *Reg. Chaotic Dynamics*, 8, 67-82 Dogru F., Tabachnikov S., (2005), Dual billiards *Math. Intelligencer*, 27(4), 18-25
- [2] Tabachnikov S., (1995): Billiards, Sc.Math.de France

## **On Some Fixed Point Results via Admissible Mappings in Quasi-Partial** *b*-**Metric Spaces**

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key-words: Quasi partial b-metric spaces,  $(\alpha, \beta)$ -admissible mappings, fixed point.

#### Abstract:

In the present work, extended fixed point theorems in metric fixed point theory are provided through admissible mappings in the context of quasi-partial b-metric spaces utilizing certain auxiliary functions, and the veracity of the outputs stated with the supplied examples is supported.

## References

- [1] Wilson, W.A., (1931): On quasi-metric spaces, American Journal of Mathematics, 53(3), 675-684.
- [2] Gupta, A., Gautam, P., (2015): Quasi-partial b-metric spaces and some related fixed point theorems, *Fixed point theory and Applications*, 2015(1), 1-12.
- [3] Alizadeh, S. A., Moradlou, F., Salimi, P., (2014): Some fixed point results for  $(\alpha, \beta) (\psi, \varphi)$ -contractive mappings, *Filomat*, 28(3), 635-647.
- [4] Liu, X. D., Chang, S. S., Xiao, Y., Zhao, L. C., (2016): Some fixed point theorems concerning  $(\psi, \varphi)$ -type contraction in complete metric spaces, J. Nonlinear Sci. Appl, 9(6), 4127-4136.
- [5] Aydi, H., Felhi, A., Sahmim, S., (2021): On fixed points in quasi partial *b*-metric spaces and an application to dynamic programming, *Thai Journal of Mathematics*, 19(2), 407-419.
- [6] Cobzaş, S., (2012): Functional analysis in asymmetric normed spaces, Birkhäuser Basel.
- [7] Reilly, I. L., Subrahmanyam, P. V., Vamanamurthy, M. K. (1982): Cauchy sequences in quasi-pseudo-metric spaces, *Monat-shefte für Mathematik*, 93, 127-140.
- [8] A. Künzi, H. P., (2001): Nonsymmetric distances and their associated topologies: about the origins of basic ideas in the area of asymmetric topology, *Handbook of the history of general topology*, 853-968.

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## **On** *q*-Generalized Hyperharmonic Numbers with Two Parameters

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key-words: q-Harmonic numbers, q-hyperharmonic numbers, Heine's binomial formula.

#### Abstract:

In this study, we introduce q-generalized harmonic numbers with two parameters  $\alpha$  and  $\beta$ ,  $H_{n,l,q}(\alpha,\beta)$  for integers n, l such that  $l \geq n$ . With the help of these numbers, we define a new family of numbers which is called q-generalized hyperharmonic numbers with two parameters  $\alpha$  and  $\beta$  of order r,  $H_{n,l,q}^{r}(\alpha,\beta)$  for integer r. Then, we consider special matrices whose entries are given by these numbers and give some matrix multiplications. One of the applications is  $M_n = E_n P_n$ , where  $n \times n$  matrices  $M_n = [m_{i,j}]$  with  $m_{i,j} = q^{-ij} H_{i,l,q}^j(\alpha,\beta)$ ,  $E_n = [e_{i,j}]$  with  $e_{i,j} = \alpha^{[i-j+1]_q} \beta^{[l-i+j-1]_q} / [i-j+1]_q - \alpha^{[i-j]_q} \beta^{[l-i+j]_q} / [i-j]_q$  for i > j,  $\alpha\beta^{[l-1]_q}$  for i = j and 0 for i < j, and q-Pascal matrix  $P_n = [p_{i,j}]$  with  $p_{i,j} = q^{j(1-i)} {i+j-1 \brack j}_q$ . Additionally, we derive some combinatorial identities for  $H_{n,l,q}(\alpha,\beta)$  and  $H_{n,l,q}^r(\alpha,\beta)$  by matrix methods.

- [1] Guo D., Chu W., (2021), Summation formulae involving multiple harmonic numbers, Appl. Anal. Discrete Math., 15, 201-212
- [2] Koparal S., Ömür N. and Çolak C. D., (2020), Some applications on q-analog of the generalized hyperharmonic numbers of order r,  $H_n^r(\alpha)$ , Hacet. J. Math. Stat., 49(6), 2094-2103
- [3] Mansour T., Shattuck M., (2014), A q-analog of the hyperharmonic numbers, Afrika Mat., 25(1), 147-160
- [4] Ömür N., Bilgin G., (2018), Some applications of the generalized hyperharmonic numbers of order r,  $H_n^r(\alpha)$ , Adv. Appl. Math. Sci., 17(9), 617-627
- [5] Ömür N., Koparal S., (2018), On the matrices with the generalized hyperharmonic numbers of order *r*, *Asian-European Journal* of Mathematics, 11(3), 1850045

## Multivariate Trigonometric Korovkin Theorem in Fuzzy Setting

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key-words: Fuzzy analysis, trigonometric Korovkin theory, Fuzzy positive linear operator.

#### Abstract:

In this presentation we generalize the trigonometric fuzzy Korovkin theorem which has been stated in [2] for k-dimensional case. In the proof of this therom, we follow an alternative way which is different from that one in [2]. In case of k=1, our theorem coincides with Theorem 5 in [2]. Also we give an example which satisfies our theorem.

- [1] Anastassiou G.A., (2005), On basic fuzzy Korovkin theory, Babes-Bolyai Mathematica Studia Univ., vol. 50, pp. 3-10
- [2] Anastassiou G.A. and Gal S.G., (2006), On fuzzy trigonometric Korovkin theory, *Nonlinear Funct. Anal. Appl., vol. 11, pp.* 385-395
- [3] Popa D., (2019), Korovkin type results for multivariate continuous periodic functions, Results Math, vol. 74, artcile number 96

## **On Square Centered Square Numbers**

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key-words: Polygonal numbers, centered polygonal numbers, pell equation.

#### Abstract:

Figurate numbers are sequence of natural numbers that can be represented by regular geometric forms. Polygonal numbers (i.e. square number) and centered polygonal numbers (i.e. centered square number) are such figurate numbers.

A square number is a natural number of the form  $S_4(n) = n^2$  and centered square number is a natural number of the form  $CS_4(n) = n^2 - n + 1$ , where  $n \in \mathcal{N}$ .

Square centered square numbers are both square and centered square numbers. In this talk I will mention about which numbers are simultaneously square and centered square numbers (i.e. square centered square numbers).

- [1] Deza E. and Deza M., (2012): Figurate numbers, World Scientific
- [2] Rosen Kenneth H., (1984): Elementary number theory and its applications, Pearson/Addison Wesley
- [3] Koshy T., (2019): Fibonacci and Lucas Numbers with Applications, John Wiley and Sons
- [4] Chandoul A., (2011): Advenced Studied ine Pure Mathematics, The Pell Equation  $x^2 Dy^2 = \pm k^2$ , 16-22. doi:10.4236/apm.2011.12005
- [5] "The on-line encyclopedia of integer sequences<sup>®</sup> (OEIS<sup>®</sup>)," Oeis.org. [Online]. Available: http://oeis.org. [Accessed: 14-May-2023].

## Estimation of the System Reliability for Poisson Ailamujia Distribution Based on Different Sampling Plans

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key-words: Rank set sampling, stress-strength, Monte Carlo simulation, efficiency.

#### Abstract:

The purpose of this study is to draw conclusions about system reliability when the stress X and strengthY distributions are independent of Poisson Ailamujia. Inference R for system reliability can be used in engineering, statistics, biostatistics, etc. It is one of the most common problems in this field. Therefore, there is a significant number of studies on this issue. Traditionally, simple random sampling (SRS) is used to estimate system reliability. However, in recent years, Ranked Set Sampling (RSS), a cheap and efficient alternative to SRS, has been used to estimate system reliability. In this study, we consider estimation of R when both stress and strength are independent Poisson-Ailamujia random variables based on RSS and Median Rank Set Sampling (MRSS). Therefore, using maximum likelihood (ML) estimates, we derive R-estimators based on SRS, RSS, and MRSS. The performances of the proposed estimators were compared to their SRS-based counterparts using a Monte Carlo simulation. The simulation results show that the proposed estimators are preferable to the SRS-based estimators in terms of efficiency.

- [1] Akgul, F. G., Senoglu, B. (2017), Estimation of using modifications of ranked set sampling for Weibull distribution, *Pakistan Journal of Statistics and Operation Research, pp. 931-958*
- [2] Esemen, M., Gürler, S. (2018) Parameter estimation of generalized Rayleigh distribution based on ranked set sample, *Journal* of Statistical Computation and Simulation, vol. 88(4), 615-628.

## The New Perspective to Fixed Point Theory on Weak Partial Metric Space

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key-words: Fixed Point, partial metric space, weak partial metric.

#### Abstract:

In 1994, S. G. Matthews [3] introduced the concept of partial metric space. In a partial metric spaces, the distance of a point in the self may not be zero. After the definition of partial metric space, Matthews proved a partial metric version of Banach's fixed point theorem. In [1, 5, 7] have been made some generalizations of the result of Matthews. Heckmann [2] introduced the concept of weak partial metric space, which is a generalized version of Matthews' partial metric space by omitting the small self-distance axiom. It is clear that every partial metric space is a weak partial metric space, but the converse may not be true. Some results for mappings in weak partial metric spaces have been obtained by [2] and [3]. In this talk, we debate and research the problem of the existence and uniqueness of some contraction type mappings on weak partial metric space.

- [1] Altun, I., Erduran, A., (2011), Fixed point theorems for monotone mappings on partial metric spaces, *Fixed Point Theory and Applications*, vol. 1, pp. 1-10.
- [2] Altun, I., Durmaz, G., (2012), Weak partial metric spaces and some fixed point results, Appl. Gen. Topol., vol. 13(2), pp. 179-191.
- [3] Durmaz, G., Acar, Ö., Altun, I., (2013), Some fixed point results on weak partial metric spaces, *Filomat, vol. 27(2), pp. 317-326.*
- [4] Heckmann R., (1999), Approximation of metric spaces by partial metric spaces, Appl. Categ. Structures, vol. 7, pp. 71-83.
- [5] Ilić, D., Pavlović, V., Rakočević, V., (2011), Some new extensions of Banach's contraction principle to partial metric space, Applied Mathematics Letters, vol. 124(8), pp. 1326-1330.
- [6] Matthews S. G., (1994), Partial metric topology, Proc. 8th Summer Conference on General Topology and Applications, *New York Acad. Sci.*, vol. 728, pp. 183-197.
- [7] Valero, O., (2005), On Banach fixed point theorems for partial metric spaces, Applied General Topology, vol. 6(2), pp. 229-240.

## A Novel Type of Inverse Soft Covering Based Rough Set and Its Application to Decision Making

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key-words: Inverse soft set, soft rough set, inverse soft rough set, decision making.

#### Abstract:

In this study, we present a new type of inverse soft covering based rough set and its basic properties. We show that our inverse soft covering upper approximation operator is smaller than with other types of inverse soft covering upper approximation operators. Moreover, we develop an algorithm following these concepts and apply it to a decision-making problem to demonstrate the applicability of the proposed methods.

- [1] Çetkin V., Aygünoğlu A. and Aygün H., (2016), A new approach in handling soft decision making problems, J. Nonlinear Sci. Appl., 9, pp. 231-239
- [2] Demirtaş N., Hussain S. and Dalkılıç O., (2020), New approaches of inverse soft rough sets and their applications in a decision making problem, *J. Appl. Math. Informatics*, 38(3-4), pp. 335-349
- [3] Feng F., Li C., Davvaz B. and Ali I.M., (2010), Soft sets combined with fuzzy sets and rough sets: a tentative approach, *Soft Computing*, 14(9), pp. 899–911
- [4] Pawlak Z., Rough sets, (1982), International Journal of Computer and Information Sciences, 11(5), pp. 341–356
- [5] Molodtsov D., (1999), Soft set theory first results, Comput. Math. Appl., 37, pp. 19-31
- [6] Zhu W. and Wang F., (2007), On three types of covering-based rough sets, *IEEE Transactions on Knowledge and Data Engineering*, 19(8), pp. 1131–1143
- [7] Zhu W., (2007), Topological approaches to covering rough sets, Information Sciences, 177(6), pp. 1499–1508

## A Note on Linear Mixed Models with Stochastic Restriction

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key-words: BLUP, MSEM, stochastically restricted linear mixed model.

#### Abstract:

The best linear unbiased predictor (BLUP) and any predictor are compared in this article using the mean squared error matrix (MSEM) criterion for a unified form of all unknown parameters in a stochastically restricted linear mixed model (SRLMM). For comparing the MSEMs of these predictors, block matrix inertias and ranks are used. For the related research in which comparable methods were used, see also [1]-[5].

- [1] Güler, N. and Büyükkaya, M. E., (2021), Rank and inertia formulas for covariance matrices of BLUPs in general linear mixed models, *Commun. Stat. -Theory Methods, vol. 50(21), pp. 4997–5012.*
- [2] Haslett, S. J. and Puntanen, S., (2010), Equality of BLUEs or BLUPs under two linear models using stochastic restrictions, Stat. Pap., vol. 51, pp. 465–475.
- [3] Ren, X., (2015), Corrigendum to "On the equivalence of the BLUEs under a general linear model and its restricted and stochastically restricted models", *Stat. Probab. Lett., vol. 104, pp. 181–185.*
- [4] Xu, J. and Yang, H., (2007), Estimation in singular linear models with stochastic linear restrictions, *Commun. Stat. Theory Methods, vol. 36(10), pp. 1945–1951.*
- [5] Yang, H., Ye, H. and Xue, K., (2014), A further study of predictions in linear mixed models, *Commun. Stat. Theory Methods*, vol. 43(20), pp. 4241–4252.

## Bezier Cubics and Neural Network Agreement Along a Moderate Geomagnetic Storm

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key-words: Bezier cubics (BC), network model (NM), total electron content (TEC), geomagnetic storm (GS).

#### Abstract:

The discussion models the IRI-2012 TEC map over a moderate geomagnetic storm period (5 days) in 2015 and compares the yield of the models. Th models are constructed with the help of Bézier cubics and machine learning. In a sense, the comparison of a classical and mechanical approach with a modern and computer-driven approach is a considerable experience for the paper. The parametric curve approach governs models of piece-wise continuous Bézier cubics, while the models employ only the TEC map. The design is separated into curve components at every five-hour curvature point and each component is handled independently. Instead of the traditional least squares method for finding control points of cubic, it utilizes the mean of every five-hour of the piece-wise curves of the TEC data. Accordingly, the prediction error can be controlled at a rate that can compete with the modern network approach. In the network model, 120 hours of the solar wind parameters and the TEC map of the moderate storm are processed. The reliability of the network model is evaluated by the correlation coefficient, mean square error, and mean absolute error is 0.9001%. The correlation coefficient (R) of the network model is 99.6%, the mean square error is 0.71958 (at epoch 47), and the mean absolute error is 0.9062%. The results agree with the literature.

## **On Generalizations of Convergence in Neutrosophic Normed Spaces**

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key-words: Neutrosophic normed space, Statistical convergence, lacunary statistical convergence.

### Abstract:

In this study, the concepts of  $\Delta^m$ -statistical convergence and  $\Delta^m$ -lacunary statistical convergence of order  $\alpha$  with respect to the neutrosophic norm in the neutrosophic normed spaces were introduced. Relations between these two types of convergence and some inclusion theorems were investigated. Additionally  $\Delta^m$ -statistically continuous functions and  $\Delta^m$ -statistical uniformly convergent functions of order  $\alpha$  were presented.

- [1] Atanassov, K., 1986, Intuitionistic fuzzy-sets, Fuzzy sets and systems, 20(1), 87-96.
- [2] Basarir, M. and Altindag, S., 2008 On  $\Delta$ -lacunary statistical asymptotically equivalent sequences, *Filomat*, 22(1), 161-172.
- [3] Braha, N., 2012, On asymptotically  $\Delta_{\lambda}^{m}$ -lacunary statistical equivalent sequences, *Appl. Math. Comput.*, 219(1), 280-288.
- [4] Cakalli, H. and Ersan, S., 2016, New types of continuity in 2-normed spaces, Filomat, 30(3), 525-532.
- [5] Et, M., Cinar, M. and Sengul, H., 2019, On delta(m)-asymptotically deferred statistical equivalent sequences of order alpha, *Filomat*, 33(7), 1999-2007.
- [6] Et, M. and Colak, R., 1995, On generalized difference sequence spaces, Soochow J. Math., 21(4), 377-386.
- [7] Fast, H., 1951, Sur la convergence statistique, Collog. Math., 2, 241-244.
- [8] Fridy, J., 1985, On statistical convergence, Analysis, 5, 301-313.
- [9] Fridy, J. and Orhan, C., 1993, Lacunary statistical convergence, Pasific J. Math., 160(1), 43-51.
- [10] Gadjiev, A. and Orhan, C., 2002, Some approximation theorems via statistical convergence, *Rocky Mountain Journal Of Mathematics*, 32(1), 129-138.
- [11] Karakus, S., Demirci, K. and Duman, O., 2008, Statistical convergence on intuitionistic fuzzy normed spaces, *Chaos Solitons Fractals*, 35(4), 763-769.
- [12] Kirisci, M. and Simsek, N., 2020, Neutrosophic metric spaces, *Mathematical Sciences*, 14(3), 241-248.
- [13] Kirisci, M. and Simsek, N., 2020, Neutrosophic normed spaces and statistical convergence, *Journal of Analysis*, 28(4), 1059-1073.
- [14] Kisi, O., 2021, Ideal convergence of sequences in neutrosophic normed spaces, *Journal of Intelligent & Fuzzy Systems*, 41(2), 2581-2590.
- [15] Kisi, O., 2021, On  $I_{\theta}$  convergence in neutrosophic normed spaces, Fundamental Journal of Mathematics and Applications, 4(2), 67-76.
- [16] Menger, K., 1942, Statistical metrics, Proc. Nat. Acad. Sci., 28(12), 535-537.
- [17] Mursaleen, M., 2000,  $\lambda$ -statistical convergence, *Math. Slovaca*, 50(1), 111-115.
- [18] Savas, E. and Gurdal, M., 2015, A generalized statistical convergence in intuitionistic fuzzy normed spaces, *Scienceasia*, 41(4), 289-294.
- [19] Smarandache, F., 2005, Neutrosophic set, a generalization of intuitionistic fuzzy sets, Int. J. Pure Appl. Math., 24, 287-297.
- [20] Zadeh, L., 1965, Fuzzy sets, Inform. Control, 8(1), 338-353.

## **On Deferred Statistical Boundedness of Generalized Difference Sequences**

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key-words: Difference sequence, deferred statistical convergence, deferred statistically cauchy.

#### Abstract:

Study of difference sequences is a recent development in the summability theory. Sometimes a situation may arise that we have a sequence at hand and we are interested in sequences formed by its successive differences and in the structure of these new sequences. Studies on difference sequences was introduced in the 1980s and then many mathematicians studied on these kind of sequences and obtained some generalized difference sequence spaces. In this paper, using the generalized difference operator  $\Delta^n$ , we introduce the concept of  $\Delta^n$ -deferred statistical boundedness and give some inclusion relations between  $\Delta^n$ -deferred statistical convergence of and  $\Delta^n$ -statistical boundedness. Our results are more general than the corresponding results in the existing literature.

- [1] Agnew, R. P., 1932, On deferred Cesàro means, Ann. of Math.(2) 33(3), 413-421.
- [2] Akbaş, K. E. and Işık, M., 2020, On asymptotically  $\lambda$ -statistical equivalent sequences of order  $\alpha$  in probability, Filomat **34**(13), 4359-4365.
- [3] Et, M. and Çolak, R., 1995, On some generalized difference sequence spaces, Soochow J. Math. 21(4), 377-386.
- [4] Et, Mikail., Bhardwaj, Vinod, K. and Gupta, S., 2022, On deferred statistical boundedness of order α. Comm. Statist. Theory Methods 51, no. 24, 8786-8798.
- [5] Fast, H., 1951, Sur la convergence statistique, Colloq. Math. 2, 241-244.
- [6] Fridy, J. A., 1985, On statistical convergence, Analysis 5(4), 301-313.
- [7] Kizmaz, H., 1981, On certain sequence spaces, Canad. Math. Bull. 24(2), 169-176.
- [8] Küçükaslan, M. and Yilmazturk, M., 2016, On deferred statistical convergence of sequences, Kyungpook Math. J. 56(2), 357-366.
- [9] Šalát, T., 1980, On statistically convergent sequences of real numbers, Math. Slovaca 30(2), 139-150.
- [10] Temizsu, F. and Et, M., 2021, Some results on generalizations of statistical boundedness, Math. Methods Appl. Sci. 44(9), 7471-7478.
# The Characteristic of Endomorphism Ring $\operatorname{End}(\mathbb{Z}_{p^m} \times \mathbb{Z}_p)$ and Its RSA Variant

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key-words: Endomorphism, ring, RSA, monoid, cryptography.

#### Abstract:

Let p be a prime number. George M. Bergman [1] found that endomorphism ring  $\operatorname{End}(\mathbb{Z}_p \times \mathbb{Z}_{p^2})$  is a semilocal ring which can not be embedded in matrices over any commutative ring. Later, Climent et al. declared that  $\operatorname{End}(\mathbb{Z}_p \times \mathbb{Z}_{p^2})$  is isomorphic to  $E_p$ , the set of two by two matrices where its first row and second row entries belong to  $\mathbb{Z}_p$  and  $\mathbb{Z}_{p^2}$  correspondingly [2]. By this research, Long D.T. et al. constructed a new RSA variant based on the monoid which its multiplication is defined same as the multiplication in  $\operatorname{End}(\mathbb{Z}_p \times \mathbb{Z}_{p^2})$  [3]. On 2016, Liu and Liu established the characteristic of endomorphism ring  $\operatorname{End}(\mathbb{Z}_p \times \mathbb{Z}_{p^m})$  for any positive integers  $m \ge 3$  [5]. Again, Long D.T. et. al. (2018) constructed a new RSA variant based on this characterization [4]. This paper describes the characterization of endomorphism ring  $\operatorname{End}(\mathbb{Z}_{p^k} \times \mathbb{Z}_p)$  for any positive integers  $k \ge$  and its RSA variant.

- [1] George M. B., (1974), Some examples in pi ring theory, Israel Journal of Mathematics, 18, 257-277
- [2] Joan J.C., Pedro N., L. Tortosa, (2011) On The Arithmetic of the endomorphism ring  $End(\mathbb{Z}_p \times \mathbb{Z}_{p^2})$ , Applicable Algebra in Engineering, Communication and Computing, 22, 91-108
- [3] Tran D.L., Tran D.T., Nguyen D.T., (2013): 2013 International Conference on IT Convergence and Security (ICITCS), pages 1-4.
- [4] Tran D.L., Le T.K.N., (2018), A Variant OF RSA Cryptosystem on The Endomorphism Ring  $End(Z_n \times Z_{n^k})$ , Hue University Journal of Science: Natural Science, 127, 1A, 101-109
- [5] Xiusheng L., Hualu L., (2016), On the arithmetic of the endomorphism ring  $End(\mathbb{Z}_p \times \mathbb{Z}_{p^m})$ , arXiv:1605.00805

# Sensitivity Analysis of a Deterministic Time-Invariant Compartmental Model

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key-words: Sensitivity analysis, compartmental models, Steady-state condition

#### Abstract:

Sensitivity analysis (SA) is commonly employed at a preliminary stage of model development process to increase the confidence in the model and its predictions by providing an understanding of how the model response variables respond to changes in the inputs, data used to calibrate it, and model structures. This talk concerns application/modification of SA techniques to compartmental models with steady-state constraint. This is explored using a deterministic, time-invariant compartmental model of global carbon cycle, and computational methods developed to maintain the initial steady-state condition are presented.

## A New Conditional Deductible in Insurance Policy

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key-words: Compensation, deductible, LER, non-life insurance, policy.

### Abstract:

Nowadays, it has been observed that the risks arising from economic and technological developments have increased. These risks are evaluated in two groups: life and non-life activities within the scope of insurance. In this study, policy features based on models related to the number and size of claims are examined, with an emphasis on non-life insurance. Existing policy modifications are explained, and then a new deductible is proposed regarding the damage occurrence condition, and LER and compensation are calculated by simulation studies in the R program.

- [1] Boucher J. P., Denuit M., and Guillén M., (2007), Risk classification for claim counts: a comparative analysis of various zeroinflated mixed Poisson and hurdle models, *North American Actuarial Journal*, 11(4), 110-131.
- [2] Henckaerts R., Côté M. P., Antonio K.,and Verbelen R., (2021), Boosting insights in insurance tariff plans with tree-based machine learning methods, *North American Actuarial Journal*, 25(2), 255-285.
- [3] Ogungbenle M. G., Adeyele J. S., and Mesioye A. E., (2020), Modeling Moments of Insurance Claim Size Under Dirac-Delta Function, *Journal of Science and Technology*, 15(1), 43-51.

## **On Quasi Modular b-Metric Spaces**

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key-words: Quasi metric space, modular metric space, fixed point

### Abstract:

This study combines the modular b-metric and quasi-modular metric structures to form a new space known as quasi-modular b-metric space. In the context of this space, some fixed-point results were additionally demonstrated.

- [1] V. V. Chistyakov, Modular metric spaces, I: Basic concepts, Nonlinear Anal., 72, (2010), 1-14.
- [2] V. V. Chistyakov, Modular metric spaces, II: Application to superposition operators, Nonlinear Anal., 72, 2010, 15-30.
- [3] S.Romeguera, P.Tirado, A characterization of Smyth complete quasi-metric space via Caristi's fixed point theorem *Fixed Point Theory Appl. (2015), 2015:183.*

## Fixed Points of Simulative Contraction on Non-Archimedean Quasi Modular b-Metric Spaces

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key-words: Quasi modular b metric space, Simulative contraction

### Abstract:

In this talk, we establish fixed point results for simulative contraction on non-Archimedean quasi modular b metric spaces. Our results generalize and extend various comparable results in the existing literature.

- [1] S. Radenovic, S. Chandok, Simulation type functions and coincidence point results, Filomat, 32:1, (2018), 141-147.
- [2] B. Samet, Best proximity point results in partially ordered metric spaces via simulation functions, *Fixed Point Theory Appl.*, 2015:32, (2015).
- [3] F. Tchier, C. Vetro, F. Vetro, Best approximation and variational inequality problems involving a simulation function, *Fixed Point Theory Appl.*, 2016:26, (2016).

# Suborbital Graphs Obtained by Different Matrix Multiplications and Fibonacci and Pell Numbers

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key-words: Suborbital graph, Fibonacci and Pell numbers, Lorentz matrix multiplication.

#### Abstract:

In this study, an element of modular group, where (u, N) = 1 and  $u^2 + (-1)^i k_i u + 1 \equiv 0 \pmod{N}$ ,  $i = 1, 2, 1 < k_i < N$  and the acts of suborbital graph constituted of with the help of this element have been scrutinized. A path directed right and left and the  $n^{th}$  vertices on this path have been achieved. These vertices have been associated Fibonacci numbers for k = 3 and Pell numbers for k = 6. Also, these vertices have been investigated with recurrence relations, continued fractions and matrix relations. Lorentz matrix that gives obtained vertices on the path directed right and left with Lorentz matrix multiplication has been obtained. Since related matrix is not an element of modular group, it has been normalized and its type has been determined. In addition, an element of modular group has been studied for (u, N) = 1 and  $u^2 + (-1)^i k_i u - 1 \equiv 0 \pmod{N}$ ,  $i = 1, 2, 1 < k_i < N$ . A new path has been attained by means of this element of modular group and Lorentz matrix multiplication. The edge conditions and the case of self-paired have been given for this path.

- [1] Değer, A. H., 2011: Curves of Minimal Length with  $\hat{Q}$  Vertex on Suborbital Graphs of the group  $\Gamma_0(n)$ , PhD. Thesis, K.T.Ü., The Graduate School of Natural and Applied Sciences, Trabzon.
- [2] Akbaba, Ü., 2016: Imprimitive Action of Minimal Length on the Suborbital Graphs of the Group  $\Gamma_0(n)$ , Master Thesis, K.T.Ü., The Graduate School of Natural and Applied Sciences, Trabzon.
- [3] Gündoğan, H. and Keçilioğlu, O., 2006, Lorentzian Matrix Multiplication and the Motions on Lorentzian Plane, *Glasnik Matematic ki, Vol. 41, 61, pp.329-334*.
- [4] Gökcan, I. and Değer, A. H., 2022, Vertices of Suborbital Graph  $F_{u,N}$  under Lorentz Matrix Multiplication, Journal of New Theory, Vol.41, pp. 35-50.

## A Hybrid Numerical Algorithm for an Inverse Problem for the General Transport Equation

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key-words: Inverse problem, a hybrid numerical algorithm, transport equation.

#### Abstract:

In this work, we focus on an inverse source problem for a general transport equation. We develope a new numerical algorithm to obtain the approximate solution of the problem. The proposed algorithm is composed of different numerical approaches which are used for the derivatives and integrals in the equation. Namely, it is based on the finite difference method, Newton Cotes formula, Lagrange polynomial approximation and the composite trapezoidal rule. This method is tested on several examples and the results show that the relative error in the computations is acceptably small. Additionally, the algorithm exhibits a significant degree of resistance against noisy data.

Solvability of some inverse problems for various transport equations is investigated in [1, 3, 4, 5, 7]. Numerical algorithms are presented in [2, 6] for the equations which don't include the absorbtion and scattering terms.

- [1] Amirov A.Kh., (2001): Integral Geometry and Inverse Problems for Kinetic Equations, Utrecht, The Netherlands, VSP
- [2] Amirov A., Ustaoğlu Z. and Heydarov B., (2011), Solvability of a two dimensional coefficient inverse problem for transport equation and a numerical method, *Transport Theory and Statistical Physics*, vol. 40(1), pp. 1-22
- [3] Bal G., Jollivet A. and Jugnon V., (2010), Inverse transport theory of photoacoustics, Inverse Problems, vol. 26(2), 025011
- [4] Cannarsa P., Floridia G., Gölgeleyen F. and Yamamoto M., (2019), Inverse coefficient problems for a transport equation by local Carleman estimate, *Inverse Problems, vol. 35(10), 105013*
- [5] Gölgeleyen F. and Yamamoto M., (2016), Stability for some inverse problems for transport equations, SIAM Journal on Mathematical Analysis, vol. 48(4), pp. 2319-2344
- [6] Gölgeleyen I., (2013), An inverse problem for a generalized transport equation in polar coordinates and numerical applications, Inverse Problems, vol. 29(9), 095006
- [7] Klibanov M.V. and Pamyatnykh S.E., (2008), Global uniqueness for a coefficient inverse problem for the non-stationary transport equation via Carleman estimate, *Journal of mathematical analysis and applications, vol. 343(1), 352-365*

## An Inverse Problem for the Kinetic Equation in an Unbounded Domain

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key-words: Uniqueness, Cauchy problem, kinetic equation, pointwise Carleman estimate, unbounded domain.

### Abstract:

In this work, we consider a kinetic equation with Cauchy data in an unbounded domain. By using the Fourier transform and the change of variables the equation is reduced to an ultrahyperbolic Schrödinger equation. By the given additional data at the initial time, uniqueness of solution of an inverse problem is investigated. Our aim is to determine the unknown function on the right-hand side of the equation and the main tool is a pointwise Carleman estimate. Carleman estimates have been used for investigating the solvability of many inverse problems for differential equations, see [1, 2, 3]. Various inverse problems for kinetic equations were studied by Amirov [4] and Anikonov [5] on a bounded domain. Numerical algorithms were developed in [6].

- [1] Klibanov M. V. and Timonov A., (2004): *Carleman estimates for coefficient inverse problem and numerical applications*, The Netherlands: VSP.
- [2] Lavrentiev M. M., Romanov V. G. and Shishatskii S. P., (1986): *Ill-Posed problems of mathematical physics and analysis*, Providence: American Mathematical Society.
- [3] Yamamoto M., (2009), Carleman estimates for parabolic equations and applications, Inverse Problems, 25, 123013.
- [4] Amirov A. Kh., (2001): Integral geometry and inverse problems for kinetic equations, Utrecht: VSP.
- [5] Anikonov Yu. E., (2001): Inverse problems for kinetic and other evolution equations, Utrecht: VSP.
- [6] Golgeleyen F. and Amirov A., (2011), On the approximate solution of a coefficient inverse problem for the kinetic equation, *Mathematical Communication*, 16, 283-298.

## **On a Second Regularized Trace Formula**

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key-words: Trace-class operator, spectrum, resolvent, regularized trace.

#### Abstract:

We consider a differential operator of Sturm-Liouville type with a bounded operator coefficient and the periodic boundary conditions. We examine the spectral properties of this operator and determine its second regularized trace formula.

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- [1] Gelfand I.M. and Levitan M.B., (1953), On a simple identity for the eigenvalues of a second-order differential operator, *Dokl. Akad. Nauk SSSR., vol. 88, no. 4, pp. 593-596.*
- [2] Lusternik L.A. and Sobolev V.I., (1975): Elements of Functional Analysis, John Wiley-Sons Inc.
- [3] Adıgüzelov E., Avcı H. and Gül E., (2001), The trace formula for Sturm-Liouville operator with operator coefficient, J. Math. Phys., vol. 42, no. 6, pp. 1611-1624
- [4] Gül E. and Ceyhan A., (2021), A Second Regularized Trace Formula for a Fourth Order Differential Operator, *Symmetry-Basel*, vol. 13, no. 4, pp. 1-17
- [5] Gül E. and Gill T.L., (2022), Regularized Trace for Operators on a Separable Banach Space, Mediterr. J. Math., vol. 19:156, pp. 1-15
- [6] Gül E., (2022), On the Regularized Trace of a Differential Operator of Sturm-Liouville Type, *Filomat, vol. 36, no. 13, pp.* 4515-4523

# **Hyperstonean Spaces**

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key-words: Hyperstonean space, perfect measure, Bochner space, Banach-Stone theorem.

#### Abstract:

For a compact Hausdorff space X, C(X) will denote the Banach space of scalar-valued continuous functions on X with the usual supremum norm. A compact Hausdorff space  $\Omega$  is called hyperstonean if  $C(\Omega)$  is a dual space, that is, the dual of a Banach space. In this study, we discuss the characterization of hyperstonean spaces and the topological structure of them.

## **Some Properties of** *q***-Durrmeyer Operators**

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q-integer, q-Durrmeyer operator, analytic function, growth estimate, operator norm.

#### Abstract:

The Bernstein polynomials are famous for their role in the advancement of the approximation theory as well as for applications in various fields. In 1930, Kantorovich introduced polynomials with respect to the Bernstein basis with integral coefficients, whose aim was to approximate integrable functions. His idea was further extended by Durrmeyer, who found polynomials of a simpler form for the same purpose [1, 3]. Along with the progress in q-calculus, numerous q-versions of the classical approximation operators have emerged. In this talk, a few q-versions of the Durrmeyer operator are overviewed [2, 4, 5, 6]. After that, we will focus on the version introduced by Gupta [4]. New results on this version will be presented.

- [1] Derriennic, M. M., 1981, Sur l'approximation de fonctions integrables sur [0,1] par des polynomes de Bernstein modifies, *J. Approx. Theory, vol. 31, pp. 325-343.*
- [2] Derriennic, M. M., 2005, Modified Bernstein polynomials and Jacobi polynomials in q-calculus, *Rend. Circ. Mat. Palermo* (*II*), *Suppl. vol.* 76, pp. 269-290.
- [3] Durrmeyer, J. L., 1967, Une formule d'inversion de la transformee de Laplace: Applications a'la theorie des moments, *These de 3e cycle, Faculte des Sciences de l'Universite de Paris*.
- [4] Gupta, V., 2008, Some approximation properties of q-Durrmeyer operators, Appl. Math. Comput., vol. 197(1), pp. 172-178.
- [5] Gupta, V. and Wang, H., 2008, The rate of convergence of q-Durrmeyer operators for 0 < q < 1, Math. Methods Appl. Sci., vol. 31(16), pp. 1946-1955.
- [6] Mahmudov, N. I., 2014. Approximation by q-Durrmeyer type polynomials in compact disks in the case q > 1, Appl. Math. Comput., vol. 237, pp. 293-303.

## **On Unbounded Order Continuous Operators 2**

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key-words: Riesz space, unbounded order convergence, unbounded order continuous operator.

#### Abstract:

Let E and F be two Archimedean Riesz spaces. A net  $(x_{\alpha})$  in E is said to be order convergent to  $x \in E$ , if there is a net  $(y_{\beta})$  in E with  $y_{\beta} \downarrow 0$  and that for every  $\beta$ , there exists  $\alpha_0$  satisfying  $|x_{\alpha} - x| \leq y_{\beta}$  for all  $\alpha \geq \alpha_0$ , and it is denoted by  $x_{\alpha} \stackrel{o}{\to} x$ . A net  $(x_{\alpha})$  in E is unbounded order convergent to  $x \in E$  if  $|x_{\alpha} - x| \wedge u \stackrel{o}{\to} x$  for all  $x \in E_+$ , and this convergence is denoted by  $x_{\alpha} \stackrel{uo}{\to} x$ . A noperator (linear map)  $T : E \to F$  is said to be unbounded order continuous (*uo*-continuous) if  $x_{\alpha} \stackrel{uo}{\to} 0$  in E implies  $Tx_{\alpha} \stackrel{uo}{\to} 0$  in F. The space of all order bounded *uo*-continuous operators is denoted by  $L_{uo}(E, F)$ . The *uo*-continuous operators have been studied by A. Bahramnezhad, K. H. Azar in [2] and by B. Turan, B. Altın, H. Gürkök in [6]. The following two open problems are given in [2].

**Problem 1** Let  $T : E \to F$  be an unbounded order continuous operator between two Riesz spaces with F Dedekind complete. Is |T| unbounded order continuous operator?

**Problem 2** Is  $L_{uo}(E, F)$  a band of  $L_b(E, F)$ ?

In this study, we investigated these problems and obtained their solutions.

## References

[1] Aliprantis CD. and Burkinshaw O.,(2006): Positive Operators, Berlin, Springer.

- [2] Bahramnezhad A. and Azar KH., (2018), Unbounded order continuous operators on Riesz spaces, Positivity, 22, 837-843.
- [3] Gao N. and Xanthos F.,(2014), Unbounded order convergence and application to martingales without probability, *Journal of Mathematical Analysis and Applications*, 415, 931-947.
- [4] Gao N., Troitsky VG. and Xanthos F.,(2017), Uo-convergence and its applications to Cesaro means in Banach lattices, *Israel Journal of Mathematics*, 220, 649-689.
- [5] Luxemburg WAJ. and Zaanen AC., (1971): *Riesz spaces I*, Amsterdam, North-Holland.
- [6] Turan B., Altın B. and Gürkök H., (2022), On unbounded order continuous operators, *Turkish Journal of Mathematics*, 46 (8), 3391-3399.

## **Stationary Acceleration Curves via Sabban Frame**

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key-words: Rigid-body motion, acceleration, curve frame.

#### Abstract:

In this study, we interpreted spherical curves with geodesic curvatures which are linear functions. We determined stationary acceleration curves by using the Sabban frame of curves. We supported the study with rigid body motions, we discussed frame motions in three dimensional Euclidean space and gave Sabban frame motion. As a result, we commented the geometric meaning of spherical curves with linear functions of geodesic curvatures.

- [1] Abazarı N., Bohner M., Sağer I. and Yaylı Y., (2017), Stationary acceleration of Frenet curves, J. of Inequa. and App., 92, DOI 10.1186/s13660-017-1354-7
- [2] Bottema O. and Roth B., (1979): Theoretical Kinematics, Dover Publications, New York
- [3] Kahveci D. and Yaylı Y., (2022), Geometric kinematics of persistent rigid motions in three-dimensional Minkowski space, Mechanism and Machine Theory, 167, 104535
- [4] Selig J. M., (2007), Curves os stationary acceleration in SE(3), IMA J. of Math. Control Inform., 24, 95-113
- [5] Selig J. M., (2013), Characterisation of Frenet-Serret and Bishop motions with applications to needle steering, *Robotica*, 31(6), 981-992
- [6] Selig J. M. and Wu Y., (2006), Interpolated rigid-body motions and robotics, *Proc. of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1086-1091
- [7] Zefran M., Kumar V. and Croke C., (1998), On the generation of smooth three-dimensional rigid body motions, *IEEE Trans. Robotics Autom.*, 12, 576-589

## **Ibragimov-Gadjiev Operators Preserving Exponential Functions**

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key-words: Ibragimov-Gadjiev operators, Modified Bernstein-type operators, Bernstein-Chlodowsky operators, Szász-Mirakjan operators, weighted approximation.

#### Abstract:

In this paper, a modification of a general linear positive operators introduced by Ibragimov and Gadjiev in 1970 is constructed. This modification preserves exponential mappings and also consists of modified Bernstein and Szász type operators, as a special case. Finally, the convergence of corresponding operators in weighted spaces is discussed.

- [1] Acar, T., Aral, A. and Gonska H., 2017, On Szász-Mirakyan operators preserving  $e^{2ax}$ , a>0, Mediterranean J. Math., 14(6): 1-14.
- [2] Aral, A., C'ardenas-Morales, D. and Garrancho, P., 2018, *Bernstein-type operators that reproduce exponential* functions ,*J. of Math. Ineq.* 12(3), 861-872.
- [3] Ibragimov, I. I. and Gadjiev, A. D., 1970: On a sequence of linear positive operators, Soviet Math. Dokl., 11(1970), 1092-1095.
- [4] Özsaraç, F., Aral, A. and Karsli, H., 2018, On Bernstein-Chlodovsky Type Operators Preserving Exponential Functions, Springer Proceedings in Mathematics & Statistics, Vol. 306, Springer, Singapore.



## Stability of Impulsive Fractional Differential Equations with Generalized Proportional Riemann-Liouville Fractional Derivatives

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key-words: Fractional differential equations, impulses, generalized proportional Riemann-Liouville fractional derivative, stability, Lyapunov functions.

### Abstract:

One of the main properties studied in the qualitative theory of differential equations is the stability of solutions. There are various types of differential equations with different initial or boundary value conditions. Recently, several types of fractional derivatives are defined, studied and applied to model the dynamics or real processes and phenomena. Stability of fractional order systems is quite recent. There are several approaches in the literature to study stability, one of which is the Lyapunov approach. However the Lyapunov approach to fractional differential equations causes many difficulties. In this paper we study differential equations with generalized proportional fractional derivative of Riemann0Liouville type. Also, impulses are involved in the equation. The applied type of fractional derivative requires a new definitions of stability excluding the initial point as well as the impulsive times. Comparison results are presented and sufficient conditions for stability are given. Examples are presented to illustrate the theory.

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## Braided Crossed Module of Lie Algebras via Tensor Product of Crossed Complexes over Lie Algebroids

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key-words: Lie algebra, algebroid, tensor product.

#### Abstract:

Chain complexes with operators are modeled as crossed complexes, which also have the non-abelian characteristics of crossed modules. Brown and Higgins introduced the tensor product of crossed complexes of groupoids [1], which is a useful tool for examining higher-dimensional algebraic structures. Tensor product of crossed complexes of algebroids[2] defined by Mosa.

The category of *R*-algebroids is given in [3]. In this work we adapt this for Lie algebras. Then we define an tensor product " $\otimes$ " for the category of crossed complexes over Lie algebras. Then, by defining a multiplication  $\theta$  for a crossed complex *C*, we will give a definition crossed differential graded algebra for Lie algebras, and show that using truncation, we obtain braided crossed modules of Lie-algebras as given in [4]. We also explore the categorical equivalences for this category.

- [1] Brown R., Higgins P.J., (1987), Tensor product and homotopies for  $\omega$ -groupoids and crossed complexes, J. Pure and Applied Algebra, 47, 1-33
- [2] Mosa G. H., (1986): Higher dimensional algebroids and Crossed complexes, PhD Thesis, University of Wales, Bangor
- [3] Mitchell B., (1972), Rings with several objects ,Ad. in Math, 8(1), 1-161
- [4] Ulualan E., (2007), Braiding for categorical and crossed Lie algebras and simplicial Lie algebras, *Turkish Journal of Mathematics*, 31(3), 239-255

# Statistical Inference for Doubly Geometric Process with the Gamma Distribution

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key-words: Geometric process, doubly geometric process, parameter estimation, lognormal distribution.

#### Abstract:

The geometric process is an important widely used stochastic monotone model in many practical applications since its introduction Lam [1]. However it has some model limitations to satisfy some requirements of the applications. Recently, the doubly geometric process more flexible than the geometric process is used as an alternative to geometric process to overcome these limitations. The process is suggested by Wu [14]. The parameter estimation problem naturally arises in the doubly geometric process applications like in the geometric process. In this study, the doubly geometric process is handled under the assumptation that the distribution of the first interarrival time is assumed to be gamma distribution distribution with parameters  $\alpha$  and  $\beta$ . The model and the distribution parameters are estimated by using the maximum likelihood method. The nonlinear optimization technique in a statistical software is used in the calculations. The asymptotic joint distribution of the estimators are investigated by a simulation study with different sample sizes and the parameter values. According to the simulation study, it can be clearly said that the estimators indicate high performance with respect to the comparison criteria.

- [1] Lam, Y., (1998), Geometric Processes and Replacement Problem, Acta Math. Appl. Sin. 4, pp. 366–377.
- [2] Wu, S., (2018), Doubly Geometric Process and Applications, *Journal of the Operational Research Society, vol.* 69 (1), pp. 66-67.

# Finitary Simplicial Complexes and Various Models from Commutative Algebra

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key-words: Finitary simplicial complexes, closability, simplicial operators.

#### Abstract:

In this talk we will analze in detail the main combinatorial properties of *simplicial complexes* on arbitrary (even infinite) ground sets, i.e. non-empty set systems closed under taking subsets. By adding a condition of *finitarity* we get a very interesting class of simplicial complexes, namely the *finitary simplicial complexes*. We analyze finitary simplicial complexes by extending or introducing various mappings associating with them suitable kinds of operators. So doing, we exhibit a cryptomorphism between the so-called *finitary simplicial operators* and the algebraic closure operators and, next, we find various equivalent conditions giving us the possibility of extending the usual cryptomorphism between matroids on a finite ground sets and Maclane-Steinitz closure operators to a specific subclass of finitary simplicial complexes. We conclude the talk describing several different examples coming from algebra, module theory and commutative ring theory.

- [1] Chiaselotti G., Infusino F.G., (2022), Locally Finite Complexes, Modules and Generalized Information Systems, *Journal of Algebra and its Applications*, 21(2), 2250033.
- [2] Chiaselotti G., Infusino F.G., (2021), Some Classes of Abstract Simplicial Complexes motivated by Module Theory, *Journal* of Pure and Applied Algebra, Volume 225, Issue 1, 106471.

# On Para-Sasaki-like Manifolds Equipped with Generalized Symmetric Metric Connection

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key-words: Almost paracontact almost paracomplex Riemannian manifold, para-Sasaki-like manifold, para-Ricci-like soliton, para-Einstein-like manifold, generalized symmetric metric connection.

#### Abstract:

This research identifies the generalized symmetric metric connection on para-Sasaki-like manifolds, which are a special class of almost paracontact almost paracomplex Riemannian manifolds. We investigate the curvature tensor, the Ricci tensor and scalar curvature tensor with regard to this connection. Para-Sasaki-like solitons on para-Sasaki-like manifolds with this connection are discussed. Finally, it is given two examples of para-Sasaki-like manifolds with this connection to illustrate the obtained results.

- [1] Golab S., (1975), On semi-symmetric and quarter-symmetric linear connections, Tensor, 29, 249-254
- [2] Manev H., Manev M., (2021), Para-Ricci-like solitons on Riemannian manifolds with almost paracontact structure and almost paracomplex structure, *Mathematics*, 9(14), 1704
- [3] Manev H., (2021) Para-Ricci-like solitons with vertical potential on para-Sasaki-like Riemannian 🗍 manifolds, Symmetry, 13, 2267
- [4] Manev H., Manev M., (2022), Para-Ricci-like solitons with arbitrary potential on para-Sasaki-like Riemannian II manifolds, Mathematics, 10(4), 651
- [5] Manev M., Staikova M., (2021), On almost paracontact Riemannian manifolds of type (n, n), J. Geom., 72, 108-114
- [6] Cao H.D., (2009), Recent progress on Ricci solitons, Adv. Lect. Math., 11, 1-38
- [7] Nagaraja H.G., Premalatha C.R., (2012), Ricci solitons in Kenmotsu manifolds, J. Math. Anal., 3(2), 18-24
- [8] Blaga A.M., (2015), Ricci solitons on para-Kenmotsu manifolds, Balkan J. Geom. Appl., 20, 1-13
- [9] Siddiqi M.D., Bahadır O., (2020), η-Ricci solitons on Kenmotsu manifold with generalized symmetric metric connection, Facta Univ. Ser. Math. Inform. 35 (2), 295-310
- [10] Bahadır O., Chaubey S., Sudhakar K., (2020), Some notes on LP-Sasakian manifolds with generalized symmetric metric connection, Honam Math. J., 42(3), 461-476
- [11] Choudhary M. A., Khedher K. M., Bahadır O., Siddiqi M. D., (2021), On golden Lorentzian manifolds equipped with generalized symmetric metric connection, *Mathematics*, 9(19), 2430
- [12] Sharfuddin S., Husain S. I., (1976) Semi-symmetric metric connexions in almost contact manifolds, Tensor, 30, 133-139
- [13] Tripathi M.M., (1999), On a semi-symmetric metric connection in a Kenmotsu manifold, J. Pure Math., 16, 67-71
- [14] Hayden H. A., (1934), Subspaces of a space with torsion, Proc. London Math. Soc., 34, 27-50
- [15] Bulut S., (2019), A quarter-symmetric metric connection on almost contact B-metric manifolds, Filomat, 33 (16), 5181-5190
- [16] Bulut S., (2021), A semi-symmetric metric connection on almost contact B-metric manifolds, Turkish J. Math., 45(6), 2455-2465
- [17] Rastogi S. C., (1978), On quarter-symmetric metric connection, C. R. Acad. Sci. Bulgar, 31, 811-814
- [18] Rastogi S. C., (1987), On quarter-symmetric metric connection, Tensor, 44(2), 133-141
- [19] Friedmann A., Schouten J. A., (1924), Uber die Geometric der halbsymmetrischen Ubertragung, Math. Z., 21, 211-223
- [20] Mishra R. S., Pandey S. N., (1980), On quarter-symmetric metric F-connections, Tensor N.S., 34, 1-7
- [21] Yano K., Imai T., (1982), Quarter-symmetric metric connections and their curvature tensors, Tensor N.S., 38, 13-18

## Jet Bundles as Whitney Sums

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key-words: 58A20, 55R25, 58A05

#### Abstract:

Jet bundles are mathematical tools that have many applications in fields such as partial differential equations, mechanics, singularity theory, and variational calculus. In addition, they have a significant role in differential geometry, where they can be used to define the tangent bundle of higher order, such as the tangent bundle of  $p^k$  velocities. There are different ways of defining jet bundles, including using an equivalence relation between local sections of a given bundle, using an equivalence relation between curves on an arbitrary manifold, or using an equivalence relation on functions from  $\mathbb{R}^p$  to M. This paper focuses on the latter type of jets, and it reviews necessary preliminary information on jet bundles and their properties, provides the geometric structure of the Whitney sum  $\oplus(TM)$ , and defines a vector space structure on each fiber of the jet bundle. The paper also demonstrates that the jet bundle can be written as a Whitney sum of *p*-tangent bundles and that this representation allows for the definition of a Riemannian metric on the jet bundle.

- Baker R, Doran C. (2002): Jet Bundles and the Formal Theory of Partial Differential Equations. In: Dorst Leo, Doran Chris, Lasenby Joan, editor. Applications of Geometric Algebra in Computer Science and Engineering. Boston, MA, USA: Birkhauser, pp. 133-143
- [2] Barco M.A., Solutions of Partial Differential Equations Using Symmetry and Symbolic Computation . PhD, La Trobe University, Australia, 2000
- [3] Bocharov A.V., Chetverikov V.N., Duzhin S.V., Khorkova N.G., Krasilshchik I.S., Samokhin A.V., Torkhov Y.N., Verbovetsky A.M., Vinogradov A.M., (1999): Symmetries and Conservation Laws for Differential Equations of Mathematical Physics, USA: American Mathematical Society
- [4] Cordero L.A., Dodson C.T.J., De Leon M., (1989): Differential Geometry of Frame Bundles, Boston, USA: Kluwer Academic Press
- [5] Dieudonne' J., (1972): Treatise on Anaysis III, New York, NY, USA: Academic Press
- [6] Ehresmann C., (1951), Les prolongements d'une vari 'et 'e diff 'erentiable, C.R. Acad. Sc. Paris, 233, 598-600
- [7] Etayo F., (1988), Lifts and isomorphisms of commutation in bundles of jets Collectanea Mathematica 39(3), 231-247
- [8] Etayo F., (1991), On jets of surfaces, Collec. Math., 42(2), 171-176
- [9] Fisher R., Laquer H.T., (1999), Second order tangent vectors in Riemannian geometry, J. Korean Math Soc, 36 959-1008
- [10] Krasilshchik J., Verbovetsky A., (2011) Geometry of Jet Spaces and Integrable Systems, J Geom Phys, 61, 1633-1674
- [11] Lee J. M., Riemannian Manifolds: An Introduction to Curvature, Springer-Verlag New York, Inc. 1991.
- [12] De Leon M., Rodrigues P.R., (1985): Generalized Classical Mechanics and Field Theory, Amsterdam, Netherlands: Elsevier
- [13] Saunders D.J., (1989): The Geometry of Jet Bundles, Cambridge-New York, NY, USA: Cambridge University Press
- [14] Wu Wenyuan, (2007): Geometric Symbolic-Numeric Methods for Differential and Algebraic Systems. PhD, The University of Western Ontario, Ontario, Canada
- [15] Krupkova O., (2000), Higher-order mechanical systems with constraints, J. Math. Phys., 41, 5304-5324
- [16] Kadioglu H., (2017), Canonical Involution on Double Jet Bundles, Turkish Journal of Mathematics, 41(4), 854-868
- [17] Kadioglu H., (2020): Prolongations of Isometric Actions to Vector Bundles, Turkish Journal of Mathematics, 44(2), 378-388
- [18] Morimoto A., (1969): Prolongation of Geometric Structures, Math. Ins. Nagoya Univ., Nagoya
- [19] Golubitskii M., Guillemin V., (1973): Stable mappings and their singularities , Springer
- [20] Sarelet W., Cantrijin F., Saunders D.J., (1995), A geometrical framework for the study of non-holonomic Lagrangian systems, J. Phys. A. Math. Gen., 28, 3253-3268
- [21] Musilová J., Hronek S., (2016), The calculus of variations on jet bundles as a universal approach for a variational formulation of fundamental physical theories, Communications in Mathematics, 24(2), 173-193

## **Bimultipliers of R-Algebroids**

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key-words: R-Algebroid, bimultiplier.

#### Abstract:

In group theory, it is well known that the action of a group on another group is determined by the automorphism group. The action of a group A on a group B is given by the homomorphism  $A \rightarrow Aut(B)$ . Any extension of group A and group B is also related to a homomorphism  $A \rightarrow Aut(B)$ . In algebra case, the action of an algebra on another is related to the multiplication algebra. In algebraic extension, the outher product takes place. The concept of multiplication algebra is defined by Maclane S. [1]. In [2], using multiplication algebras, Ege U. and Arvasi Z., introduce actor crossed modules of commutative algebras and use it to generalise some aspects from commutative algebras to crossed modules of commutative algebras. R-algebroids were especially studied by Mitchell in [3, 4, 5] and by Amgott in [6]. Mitchell gave a categorical definition of R-algebroids. Mosa on the other hand, introduced crossed modules of R-algebroids and proved their equivalence to special double algebroids with connections in [7]. Then Akca İ.İ. and Avcıoğlu O. studied on crossed modules of R-algebroids in [8], [9], [10], [11] and [12].

In this paper, firstly we introduce the set denoted Bim(M) of multipliers of an R-algebroid M, then we prove that the set is an R-algebroid called multiplication R-algebroid by defining operations on this set. By using this multiplication R-algebroid Bim(M), we define an R-algebroid morphism  $A \longrightarrow Bim(M)$ , and obtain that this morphism gives the R-algebroid action. Then we examine some of the properties associated with this action.

- [1] Mac Lane S., (1958), Extensions and Obstructures for Rings, Illinois J. Math., 121, 316-345
- [2] Arvasi Z., Ege U., (2003) Anihilators, Multipliers and Crossed Modules, Applied Categorical Structures, 11, 487-506
- [3] Mitchell B., (1972), Rings with several objects, Advances in Mathematics, 8(1), 1-161
- [4] Mitchell B., (1978), Some applications of module theory to functor categories, Bull. Amer. Math. Soc., 84, 867-885
- [5] Mitchell B., (1985), Separable algebroids, Mem. Amer. Math. Soc., 57, 333, 96.
- [6] Amgott S. M., (1986), Separable categories, Journal of Pure and Applied Algebra, 40, 1-14
- [7] Mosa G.H., (1986): *Higher dimensional algebroids and crossed complexes*, PhD Thesis, University College of North Wales, Bangor
- [8] Avcioglu O., Akça I.I., (2017), Coproduct of Crossed A-Modules of R-algebroids, *Topological Algebra and its Applications*, 5, 37-48
- [9] Avcioglu O., Akça I.I., (2018), Free modules and crossed modules of R-algebroids, *Turkish Journal of Mathematics*, 42, 2863-2875
- [10] Avcioglu O., Akça I.I., (2017), On generators of Peiffer ideal of a pre-R-algebroid in a precrossed module and applications, NTMSCI 5, No. 4, 148-155
- [11] Avcioglu O., Akça I.I., (2017), On Pullback and Induced Crossed Modules of R-Algebroids, Commun.Fac.Sci.Univ.Ank.Series A1, 66(2), 225-242
- [12] Akca I.I., Avcioglu O., (2022), Equivalence between (pre)cat<sup>1</sup>-R-algebroids and (pre) crossed modules of R-algebroids, Bull. Math. Soc. Sci. Math. Roumanie Teme, 110 (3),267-288

## Homotopies of Crossed Modules of Bimultipliers of R-Algebroids

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key-words: R-Algebroid, crossed module, homotopies, bimultipliers.

#### Abstract:

Bimultipliers of an R-Algebroids and the action of an R-Algebroid to another was given in [2]. Lavendhomme and Lucas [3] discussed the relationship between the concept of bimultiplication algebra and the crossed module structure in their work. In this study, we will define bimultipliers of an R-Algebroid crossed module  $(M, A, \eta)$  and will denote the set of bimultipliers of an R-Algebroid crossed module  $(M, A, \eta)$  and will denote the set of bimultipliers of an R-Algebroid crossed module  $(M, A, \eta)$ . Then we will prove that this set is an R-Algebroid. Homotopy between R-Algebroid crossed module morphisms was defined by Avc1oglu [1]. We will define the homotopy of bimultipliers of R-Algebroid crossed modules and we will denote the set of homotopies of bimultipliers of R-Algebroid crossed module with  $U^*(A, M)$ . Then we will prove that the set  $U^*(A, M)$  is an R-Algebroid. Additionally, we will obtain a new crossed module  $(U^*(A, M), Bim(M, A, \eta), \alpha)$  by using  $U^*(A, M)$  and  $Bim(M, A, \eta)$ . This crossed module represent the actor of the crossed module  $(M, A, \eta)$  denoted with  $\mathcal{A}(M, A, \eta)$ .

- [1] Avcioglu O., (2021), Homotopies of Crossed Modules of R-Algebroids, Applied Categorical Structures, 29(5), 827-847
- Kahriman G., (2023), Bimultipliers of R-Algebroids, 2nd International E-Conference on Mathematical and Statistical Science: A Selcuk Meeting
- [3] Lavendhomme R., Lucas T., (1996), On Modules and Crossed Modules, Journal of Algebra, 179, 936-963

# A Fixed Point Result for *T*-mean Nonexpansive Mappings in *b*-metric-like Spaces with an Application

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Key-words: Fixed point, T-mean nonexpansive mapping, b-metric like space, Fredholm integral equations, time scale.

#### Abstract:

In this study, we prove the existence and uniqueness theorem of fixed points for T-mean nonexpansive mappings in b-metric-like spaces using T-Picard iteration. As an application of our result, we demonstrate the existence and uniqueness of solutions of Fredholm integral equations on time scales. Also, we present two examples to support this result. Our results extend and improve some recent results announced in the current literature.

- [1] Alghamdi, M. A., Hussain, N. and Salimi, P., 2013, Fixed point and coupled fixed point theorems on *b*-metric-like spaces, *J. Inequal. Appl., Vol. 2013, Article 402.*
- [2] Amini-Harandi, A., 2012: Metric-like spaces, partial metric spaces and fixed points, *Fixed Point Theory Appl., Vol. 2012, Article 204.*
- [3] Bohner, M. and Lutz, D. A., 2001, Asymptotic behavior of dynamic equations on time scales, J. Differ. Equations Appl., Vol. 7, 21-50.
- [4] Bohner, M. and Peterson, A., 2001: Dynamic Equations on Time Scales, Birkhauser Boston, Berlin.
- [5] Czerwik, S., 1993, Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Ostrav., Vol. 1, 5-11.
- [6] Guseinov, G. Sh., 2003, Integration on time scales, J. Math. Anal. Appl., Vol. 285, 107-127.
- [7] Mebawondu, A. A., Izuchukwu, C., Abass, H. A. and Mewomo, O. T., 2022, Some results on generalized mean nonexpansive mapping in complete metric spaces, *Bol. Soc. Paran. Mat.*, *Vol. 40, 1-16.*
- [8] Morales, J. and Rojas, E., 2009, Some results on T-Zamfirescu operators, Revista Not. Mat., Vol. 5, No. 1, 64-71.
- [9] Zhang, S., 1975, About fixed point theory for mean nonexpansive mapping in Banach spaces, J. Sichuan Univ., Vol. 2, 67-68.

## New Estimator for the Moments of a Stochastic Control Model Based on Threshold Exceedances

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key-words: Renewal function estimation, threshold exceedances, fluctuations of maxima, stochastic control model of type (s,S).

#### Abstract:

One of the main aims of this study is to show that the extreme value theory can be used as a reliable method in statistical analysis of semi-Markovian inventory systems. This research focused on estimation problem for  $n^{th}$ ,  $n \ge 1$  order moments for a stochastic control model describing a semi-Markovian inventory model of type (s, S). It is well known that renewal processes and renewal function occurs in a natural way in inventory problems. Hence estimation of renewal function is one of the key tool of this study. There are numerous literature on estimating renewal function generated different type of distributions (see [1],[2],[3][5]). Differently current literature we consider here that demand random variables are heavy tailed with infinite variance. Therefore, we mostly focused on different estimation methods for renewal function generated by heavy tailed distributions.

The extreme value analysis is considered in two different ways according to the investigated model. These approaches are known as fluctuations of sums and fluctuations of maxima. This study mainly focused on fluctuations of maxima method. Firstly we provided a new estimator for  $n^{th}$ ,  $n \ge 1$  order moments of a semi-Markovian inventory model of type (s, s) with heavy tailed demand random variables based on threshold exceedances. We provided that the suggested estimator is consistent and asymptotically normal when n = 1. We also showed that this estimator is not consistent for  $n \ge 1$ .

- [1] Anderson, K.K., Athreya, K.B., (1987) A renewal theorem in the infinite mean case, *The Annals of Probability, vol. 88(15), pp. 388-393.*
- [2] Bebbington, M., Davydov, Y., ve Zitikis, R., (2007) Estimating the renewal function when the second moment is infinite. *Stochastic Models, vol. 23(1), pp. 27-48.*
- [3] Frees, E. W, (1986) Nonparametric renewal function estimation. Annals of Statistics, vol. 14(4), pp. 1366-1378.
- [4] Markovich N.M., Krieger, U.R., (2006) Nonparametric estimation of the renewal function by empirical data. *Stochastic Models*, vol. 22(2), pp. 175–199.

## Moment Based Approximation for a Semi-Markovian Inventory Model with Asymmetric Triangular Distributed Interference of Chance

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key-words: Semi-markovian inventory model of type (s,S), Moment based approximation, Asymmetric traiangular distribution.

#### Abstract:

In this study, approximations are proposed for a renewal reward process that describes a stochastic control model of type (s,S) with asymmetric triangular distributed interference of chance. There are various asymptotic expansions in the literature regarding probabilistic and numerical characteristics for these models ([1], [2]). Each of these investigations needs the asymptotic expansion of the renewal function generated by demand random variables. However, for some distribution families, it is difficult to derive the renewal function or it is challenging to apply the renewal function due to its complex mathematical structure. In this research simple and compact approximations are presented for the stochastic control model of type (s,S) requires only knowing the first three moments of demand random variables but not exact form of distribution function. This method based on the results of the article by Kambo et al. [3]. With this method we introduced moment based approximation for ergodic distribution function of the process X(t) describing stochastic control model of type (s,S) with triangular distributed interference of chance.

## References

[1] Feller W., (1971): Introduction to Probability Theory and Its Applications II, New York, NY: John Wiley.

- [2] Hanalıoğlu Z., Khaniyev T., (2018), Asymptotic Results for an Inventory Model of Type (s,S) with Asymmetric Triangular Distributed Interference of Chance and Delay, *Gazi University Journal of Science*, vol.31 (1), pp. 174-187.
- [3] Kambo N.S., Rangan A. and Hadji E.M., (2012), Moment based approximation to the renewal function, *Communication in Statistics-Theory and Methods*, vol.41 (5), pp. 851-868.

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# **Fixed-Circle Results via Bilateral Multi-valued Contractions**

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key-words: Bilateral contraction, metric spaces, fixed circle.

#### Abstract:

In this paper, we prove new fixed-circle results using the bilateral type multi-valued contractions on a metric space. To do this, we modify some known contractive conditions called the Jaggi-type bilateral contraction and the Dass-Gupta type bilateral contraction. A nontrivial example is provided to support the hypotheses of our main results.

- [1] Özgür N.Y. and Taş, N. (2019), Some fixed-circle theorems on metric spaces, *Bulletin of the Malaysian Mathematical Sciences* Society, vol.42(4), pp.1433-1449.
- [2] Nadler S.B. (1969), Multi-valued contraction mappings, Pac. J. Math., vol.30, pp.475-488.
- [3] Chen C.M., Joonaghany G.H., Karapınar E. and Khojasteh F. (2019), On bilateral contractions, Mathematics, vol.7:38

# A Novel Count Regression Analysis Under Asymmetric Distribution

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key-words: Count regression analysis, Zero-inflated regression analysis, symmetric statistical model.

### Abstract:

In this paper, a new count regression analysis is introduced under an asymmetric distribution. Some basic properties of the proposed regression model are studied. The maximum likelihood method is examined for estimating the unknown parameters of the regression model, and its performance is evaluated via a Monte Carlo simulation under different scenarios. The applicability of the new regression model is illustrated by real-world data analysis.

- [1] Akdoğan Y., (2022), A New Flexible Discrete Distribution with Application to Zero-Inflated Regression Analysis, *Iranian Journal of Science and Technology, Transactions A: Science, 46(4), 1219-1234*
- [2] Al-Bossly A. and Eliwa M.S., (2022), Asymmetric Probability Mass Function for Count Data Based on the Binomial Technique: Synthesis and Analysis with Inference, *Symmetry*, 14, 826.
- [3] Bakouch H., Chesneau C., Karakaya K. and Kus C., (2021), The Cos-Poisson model with a novel count regression analysis, *Hacettepe Journal of Mathematics and Statistics*, 50(2), 559-578.
- [4] Lemonte A.L., Moreno-Arenas G. and Castellares F., (2020), Zero inflated Bell regression models for count data, *J. Appl. Stat.* 47 (2), 265-286

## On the Laplacian and Laplacian-like Energies of Some Classes of Bicyclic and Tricyclic Graphs

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key-words: bicyclic and tricyclic graphs, Laplacian energy, Laplacian-like energy.

#### Abstract:

Let G be a graph with n vertices and m edges. The Laplacian matrix of the graph G is defined as L(G) = D(G) - A(G) where A(G) is the (0, 1)-adjacency matrix of graph G and the D(G) is the diagonal matrix of vertex degrees. The energy of G was defined by Gutman in [1] as

$$E(G) = \sum_{i=1}^{n} |\gamma_i|$$

where  $\gamma_1, \gamma_2, ..., \gamma_n$  are the eigenvalues of adjacency matrix of G. In chemistry, the graph eigenvalues related molecular orbital energy levels of  $\pi$  electrons in conjugated hydrocarbons. If G represents molecular graph then by Hückel orbital approximation, total  $\pi$  electron energy of conjugated hydrocarbons equal to the energy of G.[1]

In view of this, recently a Laplacian analog of E has been considered [2] and defined as:

$$LE = LE(G) = \sum_{i=1}^{n} |\mu_i - \frac{2m}{n}|$$

where  $\mu_i$ , i = 1, 2, ..., n are the Laplacian eigenvalues, m is the number of edges and n is the number of vertices of the fundamental graph.

Since Laplacian energy has some major disadvantages, J. Liu and B. Liu created the Laplacian like-energy invariant (or simply Laplacian-energy-like) LEL(G), defined as in [3]

$$LEL = LEL(G = \sum_{i=1}^{n} \sqrt{\mu_i})$$

A connected molecular graph G with n vertices and m edges is called k-cyclic if m = n - 1 + k. In particular, if k = 0, then G is called tree (acyclic) molecular graph, if k = 2, then G is called bicyclic molecular graph, if k = 3 then G is called tricyclic molecular graph. For a molecular graph G, we denote by  $\overline{G}$  the complement of a molecular graph G is a graph  $\overline{G}$  on the same vertices such that two distinct vertices of  $\overline{G}$  are adjacent if and only if they are not adjacent in G.

In this study, we give the exact expressions of LE and LEL of the following classes of bicyclic and tricyclic molecular graphs (as well as of their complements).



- [1] Gutman, (1978) The energy of a graph, *Forschungszentrum*, 103:1-22
- [2] Gutman and B. Zhou, (2006): Laplacian energy of a graph, Linear Algebra Appl., 414:29-37
- [3] Liu and Liu, (2008): A Laplacian-energy-like invariant of a graph, MATCH Commun. Math. Comput. Chem., 59:355-372

# **Approximation Process of a New Operators of Hypergeometric Type**

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karsli\_h@ibu.edu.tr key-words: Hypergeometric distribution, convergence, Korovkin theorem, modulus of continuity.

#### Abstract:

In the present paper, we construct a new sequence of operators by using the hypergeometric distribution from probability theory and obtain some approximation properties of these operators.

## References

[1] Bernstein, S. N., (1912/13). Demonstration du Théoreme de Weierstrass fondée sur le calcul des probabilités, Comm. Soc. Math. Vol: 13, 1-2.

[2] Korovkin, P. P., (1960): Linear Operators and Approximation Theory, Delhi.

[3] Lorentz, G. G., (1953): Bernstein polynomials, Toronto.

[4] Shiryayev, A. N., (1984): Probability, Springer-Verlag, NewYork.

# First Order Neutral Differential Equations with Piecewise Constant Mixed Arguments

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key-words: Neutral differential equations, piecewise constant arguments, oscillation.

#### Abstract:

Neutral differential equations with piecewise constant mixed arguments constitute a class of differential equations that incorporate both neutral delays and piecewise constant arguments. In this study, we investigate the properties and solutions of neutral differential equations with piecewise constant mixed arguments. We explore the existence and uniqueness of solutions for this class of equations. Then, we analyze the behavior and stability of the solutions by using difference equations. Moreover, for the considered equation we establish the conditions of oscillatory and convergency [1].

## References

[1] Kavgaci M.E., Bereketoğlu H. and Al Obaidi H., (2023), Some results on a first-order neutral differential equation with piecewise constant mixed arguments, *Periodica Mathematica Hungarica*, 2023, https://doi.org/10.1007/s10998-022-00512-3.

## **Statistical Inference for a New Alpha Power Topp Leone Distribution**

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key-words: Topp Leone distribution, maximum likelihood estimation, alpha power transformation.

#### Abstract:

In statistical literature, several lifetime distributions are introduced. In this study, we introduce a new two parameter life time distribution called the Alpha Power Topp Leone (APTL) distribution. We use the alpha power transformation method to obtain this distribution. The APTL distribution has a more general form than the Topp Leone (TL) distribution and for more various forms of the data sets it provides a better fit than the TL distribution. In the study, we obtain some important statistical properties for the proposed distribution such as moments, skewness, kurtosis, hazard function, survival function, and order statistics. We use the maximum likelihood (ML) estimation method to estimate the parameters of the APTL distribution. Some simulation studies are carried out to demonstrate to the applicability of the APTL distribution. In this study, we use three real data sets to illustrate the potentiality of the APTL distribution and we compare this distribution to TL, Beta, and Kumaraswamy (Ku) distributions. And from the criteria measures results, it could be said that the suggested distribution is the best candidate for the considered data sets.

- [1] Caramanis, M., Stremel, J., Fleck, and W., Daniel, S., (1983), Probabilistic production costing: an investigation of alternative algorithms, *International Journal of Electrical Power & Energy Systems, vol. 5, pp. 75-86*
- [2] Genc, A. I., Moments of order statistics of Topp-Leone distribution, Journal of Stat. Pap
- [3] Ghitany, M. E., Kotz, S., and Xie, M., (2005), On some reliability measures and their stochastic orderings for the Topp-Leone distribution, *Journal of Appl. Stat.*, vol. 32, pp. 715-722
- [4] Ghitany, M. E., (2007), Asymptotic distribution of order statistics from the Topp-Leone distribution, *International Journal of Appl. Math.*, vol. 20, pp. 371-376
- [5] Mahdavi, A. and Kundu, D., (2017), A new method for generating distributions with an application to exponential distribution, Journal of Communications in Statistics- Theory and Methods, vol. 46, pp. 6543-6557
- [6] Nadarajah S. and Kotz, S., (2003), Moments of some J-shaped distributions, Journal of Appl. Stat., vol. 30, pp. 311-317
- [7] Nigm, A. M., Essam K. A. and Zeinhum F. J., (2003), Bayesian onesample prediction of future observations under Pareto distribution, *Journal of Statistics*, vol. 37, pp. 527-536
- [8] Topp C. W. and Leone F. C., (1983), A family of J-shaped frequency functions, Journal Am. Stat. Assoc., vol. 50, pp. 209-219

## **Reversed Nabla Pachpatte Type Dynamic Inequlities**

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key-words: The nabla time scale calculus, Hardy-Copson inequality, Bennett-Leindler inequality, Pachpatte's inequality, concavity.

#### Abstract:

Novel reversed nabla Pachpatte type dynamic inequalities are obtained via concavity. These inequalities yield not only their new delta counterparts but also their new discrete and continuos versions in the special cases. Moreover these inequalities provide generalizations of nabla Bennett-Leindler type dynamic inequalities [1] when concavity does not exist.

## References

[1] Kayar Z., Kaymakçalan B. and Pelen N.N., (2021), Bennett-Leindler type inequalities for time scale nabla calculus, *Mediterr*. J. Math., vol. 18, pp. 1-18

## **Reversed Diamond Alpha Pachpatte Type Dynamic Inequlities**

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key-words: The diamond-alpha time scale calculus, Hardy-Copson inequality, Bennett-Leindler inequality, Pachpatte's inequality, concavity.

#### Abstract:

Novel reversed diamond alpha Pachpatte type dynamic inequalities are obtained via concavity. These inequalities yield not only their nabla and delta counterparts but also their discrete and continuos versions in the special cases. Moreover these inequalities provide generalizations of diamond alpha Bennett-Leindler type dynamic inequalities [1] when concavity does not exist.

## References

[1] Kayar Z., Kaymakçalan B. and Pelen N.N., (2022), Diamond alpha Bennett-Leindler type dynamic inequalities and their applications, *Math. Meth. Appl. Sci.*, vol. 45, pp. 2797–2819

# Majorization Results for Subclasses of Meromorphic *q*-Starlike Functions of Complex Order

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key-words: Univalent functions, subordination, majorization, q-differential operator, starlike functions.

#### Abstract:

In the present work, using the q-differential operator, meromorphic functions of complex order in the symmetric unit disk are defined with the help of subordination. We investigate four different majorization problems for the class of meromorphic q-starlike functions of complex order related to q-differential operator. Further, some new and well-known consequences of our main results pointed out.

- [1] Arif, M., Ul-Haq, M., Barukab, O., Khan, S. A. and Abullah, S., 2021, Majorization results for certain subfamilies of analytic functions, *Journal of Function Spaces, vol. 2021, pp. 1-6.*
- [2] Khan, N., Arif, M. and Darus, M., 2022, Majorization results for certain subfamilies of analytic functions, *Complexity, Article ID* 2385739, 6 pages.
- [3] MacGregor, T. H., 1967, Majorization by univalent functions, Duke Mathematical Journal, vol. 34(1), pp. 95-102.
- [4] Tang, H., Srivastava, H. M., Li, S. H. and Deng, G. T., 2020, Majorization results for subclasses of starlike functions based on the sine and cosine functions, *Bulletin of the Iranian Mathematical Society, vol.* 46(2), pp. 381-388.

## **Factorization of Strongly Order Bounded Operators and Their Demi Class**

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key-words: Strongly order bounded operator, b-property; pre-regular operator.

#### Abstract:

In this study, we examine factorization of the strongly order bounded operator under certain conditions. Moreover, we introduce the class of demi-stronglyorder bounded operators on a Riesz space. We study the relationship between strongly order bounded operators and demi-strongly order bounded operators. We also investigate some properties of the class of demi-strongly order bounded operators.

- [1] Aliprantis, C. D. and Burkinshaw, O., 2006: Positive Operators, Berlin, Springer.
- [2] Alpay, S., Altın, B. and Tonyalı, C., 2003, On property (b) of vector lattices positivity, 7,135-139.
- [3] Alpay, S., Altın, B. and Tonyalı, C., 2006, A note on Riesz space with property-b, *Czechoslovak Mathematical Journal*, 56(2), 765-772.
- [4] Alpay, S. and Altın, B. 2011 On riesz space with b-property and strongly order bounded operators, *Rendiconti del Circolo Matematico di Palermo*, 60(1),1-12.
- [5] Benkhaled H., Elluech, A. and Jeribi, A., 2020, The class of order weakly demicompact operators, *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas*, 114(2).
- [6] Benkhaled H., Hajji, M. and Jeribi, A., 2022, On the class of demi Dunford-Pettis operators, *Rediconti del Circolo Matematico di Palerno Series*, 2, 1-11.
- [7] Benkhaled H. and Jeribi, A., 2023, The class of demi KB-operators on Banach lattices, *Turkish Journal of Mathematics*, 47(1), 387-396.
- [8] Birnbaum D. A., 1974, Pre-regular maps between Banach lattices, Bulletin of the Australian Mathematical Society, 11, 231-254.
- Krichen, B. and O'Regan, D., 2019, Weakly demicompact linear operators and axiomatic measures of weak noncompactness, *Mathematica Slovaca*, 69(6), 1403-1412.
- [10] Machrafi, N. and Altın, B., 2022, A note on topologically b-order bounded sets and generalized b-weakly compact operators, *Hacettepe Journal of Mathematics and Statistics*, 51(2), 483-493.
- [11] Petryshyn W. V., 1966, Construction of fixed points of demicompact mappings in Hilbert space, *Journal of Mathematics Analysis and Applications*, 14(2), 276-284.

# On Novel Semigroup of Enriched Chatterjea Type Mappings in Banach Spaces

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key-words: Enriched Chatterjea type mapping, enriched Chatterjea mapping, semigroup, strong convergence, weak convergence, Mann iterative process.

#### Abstract:

Recently in [5], using the technique of enrichment of contractive mappings by Krasnoselskij averaging, Berinde and Păcurar introduced a large class of mappings, called enriched Chatterjea type mappings. The main aim of this article is to introduce a new semigroup of an enriched Chatterjea type mapping. We also establish weak and strong convergence results for enriched Chatterjea type semigroups using Mann iterative process in uniformly convex Banach spaces. Additionally, the article includes some numerical examples to illustrate the validity of our results.

- [1] Agarwal, R. P., Qin, X. & Kang, S. M. (2011). Strong convergence theorems forstrongly continuous semigroups of pseudocontractions. Appl. Math. Lett., 24:1845–1848.
- [2] Banach, S. (1922). Sur les operations dans les ensembles abstraits et leur application aux equations integrales. Fund. Math., 3, 133-181.
- [3] Berinde, V. (2019). Approximating fixed points of enriched nonexpansive mappings by Krasnoselskij iteration in Hilbert spaces. Carpathian J. Math., 35, no.3, 277-288.
- [4] Bianchini, R.M.T. (1972). Su un problema di S. Reich riguardante la teoria dei punti fissi. Boll. Un. Mat. Ital., 5, 103–108.
- [5] Berinde, V. & Păcurar, M. (2021). Approximating fixed points of enriched Chatterjea contractions by Krasnoselskij iterative method in Banach spaces. Journal of Fixed Point Theory and Applications, 23, 1–16.
- [6] Chatterjea, S.K. (1972). Fixed-point theorems. C. R. Acad. Bulgare Sci., 25, 15–18.
- [7] Ceng, L.C., Xu, H.K. & Yao, J.C. (2010). Uniformly normal structure and uniformly Lipschitzian semigroups. Nonlinear Anal., 73, 3742–3750.
- [8] Cho, S.Y. & Kang, S.M. (2011). Approximation of fixed points of pseudocontraction semigroups based on viscosity iteration process. Appl. Math. Lett., 24, 224–228.
- [9] Górnicki, J. (1989). Weak convergence theorems for asymptotically nonexpansive mappings in uniformly convex Banach spaces. Comment. Math. Univ. Carol., 30, 249–252.
- [10] Kannan, R. (1968). Some results on fixed points Bull. Calcutta Math. Soc., 10, 71-76.
- [11] Kozlowski, W. M. (2018). Monotone Lipschitzian semigroups in Banach spaces. J. Aust. Math. Soc., 105(3), 417-428.
- [12] Kesahorm, T. & Sintunavarat, W. (2020). Existence and convergence theorems concerning common fixed points of nonlinear semigroups of weak contractions. J. Fixed Point Theory Appl., 22, Art. 70, 19 pp.
- [13] Li, S., Li, L. & Su, Y. (2009). General iterative methods for one-parameter nonexpansive semigroup in Hilbert space. Nonlinear Anal., 70, 3065–3071.
- [14] Opial Z. (1967). Weak convergence of the sequence of successive approximations for nonexpansive mappings, Bull. Amer. Math. Soc., 73, 591–597.
- [15] Rhoades, B.E. (1977). A comparison of various definitions of contractive mappings. Trans. Amer. Math. Soc., 226, 257-290.
- [16] Schu, J. (1991). Weak and strong convergence to fixed points of asymptotically nonexpansive mappings. Bull. Aust. Math. Soc., 43(1), 153–159.
- [17] Suzuki, T. & Takahashi, W. (2004). Strong convergence of Mann's type sequences for one-parameter nonexpansive semigroups in general Banach spaces. J. Nonlinear Convex Anal., 5, 209–216.
- [18] Xu, H. K. (2005). A strong convergence theorem for contraction semigroups in Banach spaces. Bull. Aust. Math. Soc., 72, 371–379.
- [19] Yao, J.C. & Zeng, L.C. (2007). Fixed point theorem for asymptotically regular semigroups in metric spaces with uniform normal structure. J. Nonlinear Convex Anal., 8, 153–163.
- [20] Zhang, S. (2009). Convergence theorem of common fixed points for Lipschitzian pseudo-contraction semigroups in Banach spaces. Appl. Math. Mech. (English Edition), 30, 145–152.
## Some Contraction Mappings with Binary Relation in Partial Modular *b*-Metric Spaces

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key-words: Partial modular b-metric space,  $J_C$ -contraction mappings, binary relation.

#### Abstract:

This article seeks to develop the notation of a partial modular b-metric space by approving its topological features. Incidentally, a novel contraction mapping referred to as generalized  $J_C$ -type Suzuki contraction is acquainted, and some fixed point theorems are attested in the context of partial modular b-metric space. Finally, an example that verifies our main finding is illustrated.

- Liu, X. D., Chang, S. S., Xiao, Y. and Zhao, L. C., (2016): Some fixed point theorems concerning type contraction in complete metric spaces, J. Nonlinear Sci. Appl., 9, 4127-4136.
- [2] Kesik, D., Büyükkaya, A. and Öztürk, M., (2023): On modified interpolative almost type contraction in partial modular *b*-metric spaces, *Axioms, (submitted)*.
- [3] Hosseinzadeh, H., Parvaneh, V., (2020): Meir-Keeler type contractive mappings in modular and partial modular metric spaces., *Asian-Eur. J. Math.*, 13, 1-18.
- [4] Das, D., Narzary, S., Singh, Y. M. and Khan, M. S., (2022): Fixed point results on partial modular metric space, *Axioms*, *11*, 62.
- [5] Shukla, S., (2014): Partial metric spaces and fixed point theorems, Mediter. J. Math., 11(2), 703-711.
- [6] Ege, M. E., Alaca, C., (2018): Some results for modular metric spaces and an application to system of linear equations, *Azerbaijan J. Math.*, 8(1), 3-14.
- [7] Sawangsup, K., Sintunavarat, W. and Roldan Lopez de Hierro, A. F., (2017): Fixed point theorems for contractions with applications to the solution of nonlinear matrix equations, *J. Fixed Point Theory Appl.*, 19, 1711-1725.
- [8] Al-Sulami, H. H., Ahmad, J., Hussain, N. and Latif, A., (2018): Relation theoretic contraction results with applications to nonlinear matrix equations, *Symmetry*, *10*, 767.

# On Quaternions whose Components are Higher Order Generalized Fibonacci Numbers

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key-words: Quaternions, higher order generalized Fibonacci quaternions, recurrence relation, generating functions.

### Abstract:

In this talk, 3-parameter higher order quaternions are introduced with the help of the higher-order generalized Fibonacci numbers. This definition includes not only one-parameter, two and three-parameter quaternions, but also split quaternions, semi quaternions, and 1/4 quaternions. We give some algebraic properties of these types of quaternions.

- [1] Şentürk T.D., Ünal Z., (2022), 3-Parameter Generalized Quaternions, Comput. Methods Funct. Theory., vol.22, pp. 575-608.
- [2] Kizilates C., Kone T., (2021), On Fibonacci quaternions, J. Anal., vol.29, pp.1071-1082.
- [3] Pashaev O.K., (2021), Quantum calculus of Fibonacci divisors and infinite hierarchy of bosonic-fermionic golden quantum oscillators, *Internat. J. Geom. Methods Modern Phys.*, vol.18(5), 2150075.

## A Novel Computational Scheme for Numerical Solution of the Regularized Long Wave Equation

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key-words: Regularized long wave equation, nonic B-spline, collocation method.

#### Abstract:

The Regularized Long Wave (RLW) equation

#### $u_t + u_x + \varepsilon uu - \mu u_{xxt} = 0$

was first explored by Peregrine [1] to model the propagation of weakly nonlinear and dispersive waves. Since the analytical solutions of the RLW equation are only available for restricted solution set of initial and boundary conditions, obtaining the numerical solutions of the RLW equation has increasingly become important over the last years. In the present study, a new numerical scheme is introduced to get the approximate solutions of the RLW equation. This scheme is created by using nonic B-spline collocation procedure in the spatial discretization and fourth-order two step scheme in the temporal discretization. As a test problem, the motion of the single solitary wave is studied. To show the accuracy and efficiency of the proposed scheme, the  $L_{\infty}$  error norm and three invariant constants are computed and compared with the existing techniques in the literature. The obtained results show that the present scheme exhibits high accuracy in obtaining the numerical solutions of the RLW equation. Also, the temporal order of convergence is computed and seen to be compatible with its theoretical value.

- [1] Peregrine D.H., (1966), Calculations of the development of an undular bore, *Journal of Fluid Mechanics, vol. 25, pp. 321-330.*
- [2] Irk D. and Keskin P., (2017), Quadratic trigonometric B-spline Galerkin methods for the regularized long wave equation, Journal of Applied Analysis and Computation, vol. 7, pp. 617 - 631.
- [3] Zaki S.I., (2001), Solitary waves of the splitted RLW equation, Computer Physics Communications, vol. 138, pp. 80-91.

## **Bivariate Operators that Reproduce Exponential Functions**

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key-words: Bivariate operators, Korovkin type theorem, modulus of continuity, Voronovskaya-type theorem.

#### Abstract:

The bivariate form of generalized Bernstein operators that reproduce exponential functions will be constructed. We then examine some results for the approximations of these operators. We demonstrate a Korovkin-type convergence theorem using test functions. Next, we demonstrate the convergence rate using the continuity modulus and present a theorem of the Voronovskaya type. Finally, we contrast how bivariate operators converge.

- [1] Chen, X., Tan, J., Liu, Z. and Xie, J., 2017, Approximation of functions by a new family of generalized Bernstein operators. J. Math. Anal. Appl. 450, 244–261.
- Mursaleen, M., Ansari, K. J. and Khan, A., 2016, On (p, q)-analogue of Bernstein operators. Appl. Math. Comput. 266, 874-882 (2015); Erratum: Appl. Math. Comput. 278, 70-71.
- [3] Mohiuddine, S. A., Acar, T. and Alotaibi, A., 2017, Construction of a new family of Bernstein-Kantorovich operators. Math. Meth. Appl. Sci. 40, 7749–7759.
- [4] Deshwal, S., Ispir, N. and Agrawal, P. N., 2017, Bivariate operators of Bernstein-Kantorovich type on a triangle, Appl. Math. Inf. Sci., 11(2), 423–432.
- [5] Pop, O. T. and Farcas, M. D., 2009, About the bivariate operators of Kantrovich type, Acta Math. Univ. Comenianae Vol. LXXVIII, 1, 43-52.

# Radii of the Lemniscate Starlikeness and Convexity of the Functions Including Derivatives of Bessel Functions

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key-words: Normalized Bessel functions of the fist kind, lemniscate starlikeness, lemniscate convexity, radius problem.

#### Abstract:

In this work, our aim is to determine the radii of starlikeness and convexity associated with lemniscate of Bernoulli for three different kinds of normalizations of the function  $N_{\nu}(z) = az^2 J''_{\nu}(z) + bz J'_{\nu}(z) + cJ_{\nu}(z)$ , where  $J_{\nu}(z)$  is the Bessel function of the first kind of order  $\nu$ . The key tools in the proof of our main results are the Mittag-Leffler expansion for the function  $N_{\nu}(z)$  and properties of real zeros of it. Also, we give tables and figures related with special cases of parameters.

- [1] Aktaş, İ., 2020, Lemniscate and exponential starlikeness of regular Coulomb wave functions, *Studia Scientiarum Mathematicarum Hungarica, vol. 57(3), pp. 372-384.*
- [2] Kazımoğlu, S. and Deniz, E., 2022, Radius Problems for Functions Containing Derivatives of Bessel Functions. Comput. Methods Funct, Theory, doi.org/10.1007/s40315-022-00455-3.
- [3] Kazımoğlu, S. and Deniz, E., 2022, The radii of starlikeness and convexity of the functions including derivatives of Bessel functions, *Turkish Journal of Mathematics, vol.* 46(3), pp. 894-911.
- [4] Madaan, V., Kumar, A. and Ravichandran, V., 2020, Radii of Starlikeness and Convexity of Some Entire Functions, Bull. Malays. Math. Sci. Soc., vol. 43, pp. 4335-4359.
- [5] Mercer, A. M. D., 2022, The zeros of  $az^2 J''_{\nu}(z) + bz J'_{\nu}(z) + cJ_{\nu}(z)$  as functions of order, Int. J. Math. Math. Sci., vol. 15(2), pp. 319-322.

## **On The Parity of r-Derangement Numbers**

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key-words: Derangement numbers, r-derangement numbers, odd numbers, even numbers, omega invariant.

#### Abstract

There are many integer sequences having a recurrance relation in Number Theory and the set of the numbers of derangements is one of them. In this study, we focused on the generalized form of these numbers called *r*-derangements. We give some properties of them including their parity and some characteristics of each term.

- [1] Aldous J. M., Wilson R. J., (2004): Graphs and Applications, An Introductory Approach, Springer, London
- [2] Delen S., Cangul I. N., (2019), Extremal Problems on Components and Loops in Graphs, Acta Mathematica Sinica, English Series, 35(2), 161-171
- [3] Delen S., Cangul I.N., (2018), A New Graph Invariant, Turkish Journal of Analysis and Number Theory, 6(1), 30-33
- [4] Demirci M., Ozbek A., Akbayrak O., Cangul I.N., (2021), Lucas Graphs, Journal of Applied Mathematics and Computing, 65 (1-2), 93-106
- [5] Demirci M., Cangul I.N., (2020): Tribonacci Graphs, The ITM Web of Conferences, 34, 01002, Third ICAMNM 2020
- [6] Duran O., Omur N., Koparal S., (2020), On sums with generalized harmonic, hyperharmonic and special numbers, *Miskolc Mathematical Notes*, 21 (2), 791–803
- [7] Gertsch A., (1997), Generalized harmonic numbers, Proc. Royal Acad. Sci. Paris, Ser. I Math., 324 (1), 7-10
- [8] Koparal S., Omur N., Sudemen K. N., (2022), Some Identities for Derangement Numbers, *Miskolc Mathematical Notes*, 23, 773-785
- [9] Macmahon P. A., (1915-1916), Combinatorial Analysis, Cambridge University Press
- [10] Yurttas Gunes A., Delen S., Demirci M., Cevik A.S., Cangul I.N., (2020), Fibonacci Graphs, Symmetry-Basel, 12, 1383

# The Images of Quasi-Sober Diframes Under Certain Morphisms

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key-words: Quasi-sobriety, hdiFrm morphism.

#### Abstract:

Sobriety is an essential concept in point-free topology due to the equivalence between the category of sober spaces and the category of spatial frames. When making generalizations it is often necessary to consider whether the resulting loss of information is significant. In the context of frame theory, the aforementioned equivalence between topological spaces and frames has provided the verification of the generalization. Quasi-sobriety is a concept that has been defined with the aim of broadening the scope of the theory of diframes and creating a more inclusive and comprehensive generalization. Through our study, we provide results on the preservation of quasi-sobriety under certain morphisms of the category of diframes, contributing to a deeper understanding of the underlying structures of point-free topology and the importance of sobriety in the theory of diframes.

- [1] Picado J. and Pultr A., (2012): Frames and locales. Topology without points, Springer Basel AG
- [2] Korkmaz E. and Ertürk R., (2018) On a new generalization of ditopological texture spaces, *Journal of Intelligent & Fuzzy* Systems, vol. 35, pp. 5529-5539
- [3] Korkmaz E. and Ertürk R., (2018) Separation Axioms in Diframes, *European Journal of Pure and Applied Mathematics, vol.* 11, pp. 612-627

## On the Algebra of Commutative Quaternions and Hamilton Matrices

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key-words: Quaternion, Commutative Quaternion Algebra, Hamilton Operators.

#### Abstract:

The  $q = a_0e_0 + a_1e_1 + a_2e_2 + a_3e_3$  a real quaternion which has four bases containing  $e_0 = 1, e_1, e_2, e_3$  and has four real numbers sequential  $a_0, a_1, a_2, a_3$  which have the features

 $e_1^2 = e_2^2 = e_3^2 = -1$   $e_1 \times e_2 = e_3, e_2 \times e_3 = e_1, e_3 \times e_1 = e_2,$  $e_2 \times e_1 = -e_3, e_3 \times e_2 = -e_1, e_1 \times e_3 = -e_2,$ 

was defined by Hamilton in 1843. Let be *IH* set of real quaternions given as an expansion of 4-dimensional complex numbers {  $IH, \bigoplus, \mathbb{R}, +, \cdot, \odot, \times$ } system is an algebra of quaternions. Hamilton describing quaternions has introduced a new multiplication operation into vector algebra, so that division for two vectors is also possible. With the definition of a new multiplication on the set of quaternions, the examination of motions in Euclidean space has been simplified. In this study, the commutative quaternions defined by C. Segre in 1892 are discussed. The general form of commutative quaternions are elliptical quaternions. A special case of elliptical quaternions are real quaternions. The algebraic structure of the commutative quaternions defined on the field of real numbers is constructed and the basic definitions and operations on this kind of quaternions are given. The quaternion multiplication of commutative quaternions defined in [1] has been expressed and with the help of a linear operator; has been expressed in matrix multiplication form. It has been seen that the matrix multiplication operators obtained are similar to the Hamilton operators and the properties of this kind of quaternions have been investigated.

- [1] Aristidou, M., (2009), A note on quaternion rings, International Journal of Algebra, 3(15), 1-15.
- [2] Adler, A., Coury, J.E., (1995): The Theory of Numbers, Jones and Bartlett Publishers, Boston.
- [3] Hacisalihoğlu, H.H., (1983), Motion Geometry and Quaternions Theory, *Gazi University, Faculty of Arts and Sciences Publications Mathematics*, 2.
- [4] Agrawal, O.P., (1987), Hamilton operators and dual number-quaternions in spatial kinetics, *Mechanisms and Machine Theory*, 22, 569-575.

# **Omega Invariant of Second and Third Power Graphs of Tadpole Graphs**

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key-words: Omega invariant, tadpole graph, power graph

#### Abstract:

Tadpole graphs are one of the classical graph classes with different properties than other similar classes such as path, cycle, star, complete graphs etc. k-th power graph of a graph is obtained from the graph by adding a new edge between all pairs of vertices with distance maximum k. Omega invariant is a new topological and combinatorial graph invariant defined recently. In this work, we study the second and third power graphs of the tadpole graphs and obtain several properties of them. Especially, the omega invariant and the number of edges of the second and third power graphs of tadpole graphs are obtained. The results given here have many combinatorial properties and applications.

- [1] Delen S., Cangul I.N., (2019), Extremal Problems on Components and Loops in Graphs, Acta Mathematica Sinica, English Series, 35(2), 161-171
- [2] Delen S., Cangul I.N., (2018), A New Graph Invariant, Turkish Journal of Analysis and Number Theory, 6(1), 30-33

## **Classification of Color Spaces in Various Combinations Using Deep Learning Models**

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key-words: Convolutional neural network, covid-19 detection, pneumonia detection, X-ray chest image.

#### Abstract:

The Covid-19 virus has spread worldwide as a pandemic. Scientists have made great efforts to reduce these problems and find solutions. The Reverse Transcription-Polymerase Chain Reaction (RT-PCR) test used for detection had a low success rate and required time to obtain the test. Therefore, deep learning models were used successfully in many areas as an auxiliary detection method for the diagnosis of the disease, and X-Ray was used to identify infected and normal individuals in lung X-ray images. Since the symptoms of Covid-19 are similar to pneumonia, many deep learning models have been proposed for the detection of the disease. In this study, the detection of Covid-19 and pneumonia on X-Ray images is performed. A simple Convolutional Neural Network (CNN) model is aimed to be used for the detection of diseases. To show that the model achieved successful classification regardless of the dataset properties, BGR - HSV - HSL, BGR - HSV - HSL - RGB, and BGR - HSV - HSL - RGB - YUV color space transformations were applied to X-ray images. The binary classification (Covid-19 or pneumonia) accuracy rate in the X-ray images for each color combination is 0.989, 0.981, 0.995, and the F1 score is 0.993, 0.988, 0.997, respectively. Precision, Recall, Specificity, and Mean Squared Error metrics are calculated separately for each color space. In the proposed model, the best performance is achieved with the color space combination: BGR - HSV - HSL - RGB - YUV, according to the metric results.

- [1] Ucar E., Atila U., Ucar M. and Akyol K., (2021), Automated detection of Covid-19 disease using deep fused features from chest radiography images, *Biomed Signal Process Control, vol. 69*
- [2] Nayak S. R., Nayak D. R., Sinha U., Arora V. and Pachori R. B., (2021), Application of deep learning techniques for detection of COVID-19 cases using chest X-ray images: A comprehensive study, *Biomed Signal Process Control*, vol. 64
- [3] Haque K. F., Haque F. F., Gandy L. and Abdelgawad A., (2020), Automatic Detection of COVID-19 from Chest X-ray Images with Convolutional Neural Networks, 020 International Conference on Computing, Electronics and Communications Engineering, pp. 125–130

## **Generalized Closure Operators on Bounded Lattices**

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key-words: Implication, uninorm, closure operator, order, bounded lattice.

#### Abstract:

In this paper, closure operators obtained from functions having some properties on bounded lattices are defined. By this way, it is aimed to generalize the notions T and U-closure operation which are obtained from t-norms and uninorms, respectively. The relationships between the closure operators obtained from functions and their  $\phi$ -conjugacies (N dual functions) are determined. In special cases, the closure operators from implications on bounded lattices are defined and the conditions making closure operators from R-, (S,N)-, QL- implications are determined. Also, the closures of sets according to the closure operators from implications are introduced and the properties are discussed.

- [1] Baczyński M., Jayaram B., (2008): Fuzzy Implications, Studies in Fuzziness and Soft Computing, vol. 231, Springer, Berlin, Heidelberg
- [2] Birkhoff G., (1967): Lattice Theory, 3 rd edition, Providence
- [3] Burris S., Sankappanavar H.P., (2011): A Course in Universal Algebra, New York, Springer-Verlag
- [4] Ince M. A., Karacal F., (2019), t-closure Operators, International Journal of General Systems, 48, 139-156
- [5] Karacal F., Koroglu T., (2022), A Principal Topology Obtained From Uninorms, Kybernetika, 58, 6, 863-882
- [6] Klement E.P., Mesiar R. and Pap E., (2000): Triangular Norms, Kluwer Academic Publishers, Dordrecht
- [7] Ma Z., Wu W.M., (1991), Logical Operators on Complete Lattices, Information Science, 55, 77-97

## Lototsky-Chlodowsky Operators

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key-words: Lototsky-Chlodowsky operators, Korovkin-type approximation theorem, weighted space.

#### Abstract:

The aim of the present paper is to introduce Lototsky-Chlodoowsky operators and aproximating properties in weighted space of these operators.

- [1] King.J.P.,(1966), The Lototsky transform and Bernstein polynomials, Can.J.Math.18,89-91.
- [2] Abel,U., Agratini,O., (2021), On the Durrmeyer-Type Variant and Generalizations of Lototsky-Bernstein Operators,Symmetry,13,1841.
- [3] Xu,X.W., Zeng, X.M., Goldman, R., (2017), Shape preserving properties of univariate Lototsky-Bernstein operators, J.Approx. Theory 224, 13-42.
- [4] Popa,D., (2020),Intermediate Voronovskaja type results for the Lototsky-Bernstein Operators,RACSAM. 114,12.
- [5] Jakimovski, A. (1959), A generalization of the Lototsky method of summability, Michigan Math. J., 277-290.

## A Mathematical Modeling of Post-Myocardial Infarction

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key-words: Myocardial infarction, mathematical modeling, inflammation, macrophages, heart cell.

#### Abstract:

A limited amount of regeneration and repair can occur in the heart. One of the main causes of morbidity and mortality globally is myocardial infarction (MI), which is characterized by ongoing cell death and a correspondingly decreased capacity of the heart to repair itself. A new mathematical model for investigating left ventricular (LV) remodeling and related events following MI is presented. In the absence of medical interventions, the model adequately characterizes and predicts the interactions between heart cells and the immune system after MI. To show how well the new model works and performs, the resulting system of nonlinear ordinary differential equations is examined analytically and numerically.

## **Trigonometric Cubic Bezier Transition Curves**

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key-words: Bézier curve, trigonometric basis functions, transition curve.

#### Abstract:

Curves and surfaces design is an important topic of CAGD (Computer Aided Geometric Design ) and computer graphics. The parametric representation of curves and surfaces specially in polynomial form is most suitable for design, as the planar curves cannot deal with infinite slopes and are axis dependent too. The theory of Bezier curves uphold a key position in CAGD. In recent year trigonometric polynomial curves like those of Bezier type are considerably in discussion. Many new curves related with Bezier curves are introduced by many authors. In this paper, we present transition curves from line to circle and circle to circle defined by this new type curve with shape parameters.

- Huayong L., Lu L.and Daming Z., (2011), Study on a Class of TC-Bezier Curve with Shape Parameters, Journal of Information & Computational Science, 8(7) 1217-1223
- [2] Han X. A., Ma Y.C.and Huang X.L., (2009), The cubic trigonometric Bezier curve with two shape parameters, Applied Mathematics Letters, 22, 226-231
- [3] Han X., (2002), Quadratic trigonometric polynomial curves with a shape parameter, *Computer Aided Geometric Design*, 19, 503-512
- [4] Farin G., (2002): Curves and Surfaces for CAGD, A Practical Guide, Margan Kaufmann, 5th edition
- [5] Habib Z. and Sakai M.,.(2005), Spiral Transition Curves and Their Applications, *Scientiae Mathematicae Japonicae*, 61(2), 195-206
- [6] Saxena A. and Sahay B., (2005): Computer Aided Engineering Design, Springer

## Some Important Operators on the Space of Discrete-Time Interval Signals

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key-words: Inner-product quasilinear space, interval-valued function, discrete-time interval signal, fourier transform, autocorrelation function.

#### Abstract:

In electrical engineering, the notion of signal takes an important place in many applications. A signal is defined as a function from a subset of  $\mathbb{R}$  into  $\mathbb{C}$ . In many real world problems, the value of a signal at any time may not be completely known. The concept of discrete-time interval signal provides a suitable way to analyses such these signals. In this study, we present some operators that have an important place in the processing of signals with inexact data, e.g., Fourier transform and autocorrelation function. Our results give approximate informations about the Fourier transform and autocorrelation function of discrete-time interval signals.

- [1] Alefeld G. and Herzberger J., (1974): Einführung in die Intervallrechnung, Bibliographhisches Institut., Mannheim.
- [2] Alefeld G. and Herzberger J., (1983): Introduction to Interval Computations, Academic Press, New York.
- [3] Aseev S.M., (1986), Quasilinear operators and their application in the theory of multivalued mappings, *Proceedings of the Steklov Institute of Mathematics*, 2, 23-52.
- [4] Aubin J.P. and Frakowska H., (1990): Set-Valued Analysis, Birkhauser, Boston.
- [5] Crouzet J.F. and Strauss O., (2010), Interval-valued probability density estimation based on quasi-continuous histograms: Proof of the conjecture, *Fuzzy Sets and Systems*, 183, 92-100.
- [6] Graba G. and Strauss O., (2016), An interval-valued inversion of the non-additive interval-valued F-transform: Use for upsampling a signal, *Fuzzy Sets and Systems*, 288, 26–45.
- [7] Jaulin L., Kieffer M., Didrit O. and Walter E.,(2001), Applied Interval Analysis with Examples in Parameter and State Estimation, *Robust Control and Robotics*.
- [8] Kulisch U., (1969): Grundzüge der Intervallrechnung, in: Jahrburch Überblicke Mathematik. Bibliographhisches Institut., Mannheim.
- [9] Levent H. and Yılmaz Y., (2017), An Application: Representations of some systems on non-deterministic EEG signals, J. Biostat Biometric. App., 1(2), 101.
- [10] Moore R.E., Kearfott R.B. and Cloud M.J., (2009): Introduction to Interval Analysis, SIAM, Philadelphia.
- [11] Yılmaz Y. and Levent H., (2021), Inner-product quasilinear spaces with applications in signal processing, *Advanced Studies: Euro-Tbilisi Mathematical Journal*, 4, 14.
- [12] Levent H. and Yılmaz Y., (2022), Analysis of signals with inexact data by using interval-valued functions, *Journal of Analysis*, 10.1007/s41478-022-00422-0.
- [13] Levent H. and Yılmaz Y., (2023), Fourier Transform of Interval Sequences and Its Applications, Journal of Intellegent & Fuzzy Systems, Accepted.
- [14] Rico A. and Strauss S. (2008), Imprecise expactations for imprecise linear filtering, *International Journal of Approximate Reasoning*.
- [15] Strauss O. and Rico A. (2012), Towards interval-based non-additive deconvolution in signal processing, *Soft Comput.*,16, 809-820.
- [16] Vetterli M., Kovacevic J., and Goyal V.K. (2014): Foundations of Signal Processing.

## An Approach Structure Related with Fell Topology

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key-words: Fell Topology, lower regular function frame, approach Space.

#### Abstract:

Hyperspaces of topological spaces are an important way of obtaining information on the structure of topological space. The Fell topology is a useful construct in terms of applications, especially in convex analysis, probability theory and its applications to optimization [1,2]. In this paper, we consider an Hausdorff approach space, then we construct a new approach structure in the setting of hyperspaces. We define the new structure by means of lower regular function frames defined in [3] and proved that the topological coreflection of this new structure is the ordinary Fell topology.

- [1] Beer, G., Kenderov, P. (1988): On the arg min multifunction for lower semicontinuous functions *Proc. Amer. Math. Soc., vol.* 102, 107-113
- [2] Beer, G., Luchetti, R:, (1991): Convex optimization and epi-distance topology Trans. Amer. Math. Soc. vol 327, 795-813
- [3] Lowen, R. (2015): Index Analysis, Approach Theory at Work, Springer

## **Generalized Framed Helices in Euclidean 3-Space**

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key-words: Framed curve, helix, slant helix, clad helix, k-slant helix, singularity.

#### Abstract:

In this study, we introduce framed k-slant helices from the viewpoint of singularity theory in Euclidean-3 space. Framed k-slant helices are regular or singular curves in Euclidean-3 space. Especially, the family of framed k-slant helices represents a generalized version of framed helices, framed slant helices, framed clad helices in the category of framed curves, as well as helices, slant helices, clad helices, g-clad helices in the category of regular curves. We construct successive alternative adapted frames to define the framed k-slant helix and express its axis. We obtain a characterization of the framed (k+1)-slant helix in terms of the k-alternative adapted frame. Moreover, we discover interesting relationships between the framed k-slant helix and its generalized spherical images with respect to the adapted frame. Finally, we present attractive visualizations of the framed helix, framed slant helix, and framed clad helix as regular curves or smooth curves with singular points.

- [1] Ali, A.T., (2012), New special curves and their spherical indicatrix, Glob. J. Adv. Res. Class. Mod. Geom., 1(2), 28-38
- [2] Ali, A.T., (2017), Generalization of general helices and slant helices, Journal of Mahani Mathematical Research Center, 6(1), 25-41
- [3] Honda, S., Takahashi, M., (2016), Framed curves in the Euclidean space, Advances in Geometry, 16(3), 265-276
- [4] Honda, S., (2018), Rectifying developable surfaces of framed base curves and framed helices, Advanced Studies in Pure Mathematics, 78, 273-292
- [5] Honda, S., Takahashi, M., (2020), Bertrand and Mannheim curves of framed curves in the 3-dimensional Euclidean space, *Turkish Journal of Mathematics*, 44(3), 883-899
- [6] Izumiya, S., Takeuchi, N., (2004), New special curves and developable surfaces, Turkish Journal of Mathematics, 28(2), 153-164
- [7] Kula, L., Yaylı, Y., (2005), On slant helix and its spherical indicatrix, Appl. Math. Comput., 169(1), 600-607
- [8] Kula, L., Ekmekci, N., Yaylı, Y., İlarslan, K. (2010), Characterizations of slant helices in Euclidean 3-space, Turkish Journal of Mathematics, 34(2), 261-274
- [9] Li Y., Wang, Z. Zhao, T., (2020), Slant helix of order n and sequence of Darboux developables of principal-directional curves, Math Meth Appl Sci., 43, 9888-9903
- [10] Li Y., Tuncer O.O., (2023), On (contra)pedals and (anti)orthotomics of frontals in de Sitter 2-space, Math. Meth. Appl. Sci., 46(9), 11157-11171
- [11] Mak, M., (2023), Framed Clad Helices in Euclidean 3-Space, Filomat. (Accepted)
- [12] Okuyucu, O.Z., Canbirdi, M., (2021), Framed slant helices in Euclidean 3-space, Advances in Difference Equations 2021(1), 1-14
- [13] Ramis, Ç., Yılmaz, B., Yaylı, Y., (2022), New Associated Curves k-Principle Direction Curves and N<sub>k</sub>-Slant Helix, Hagia Sophia Journal of Geometry, 4(2), 19-27
- [14] Takahashi, M., (2016), Legendre curves in the unit spherical bundle over the unit sphere and evolutes, Contemporary Mathematics, 675, 337-355
- [15] Takahashi, T., Takeuchi, N., (2014), Clad helices and developable surfaces, Bull, Tokyo Gakugei Univ. Nat. Sci., 66, 1-9
- [16] Takahashi, T., (2019), The Generalization of Helices, Caspian Journal of Mathematical Sciences (CJMS), 8(2), 178-195
- [17] Tuncer, O.O., Ceyhan, H., Gök, İ., Ekmekci, F. N., (2018), Notes on pedal and contrapedal curves of fronts in the Euclidean plane, *Math. Meth. Appl. Sci.*, 41(13), 5096-5111
- [18] Uzunoğlu, B., Gök, İ., Yaylı, Y., (2016), A new approach on curves of constant precession, Appl. Math. Comput., 275, 317-323
- [19] Wang, Y., Pei, D., Gao, R., (2019), Generic properties of framed rectifying curve, Mathematics, 7(1)
- [20] Yao, K., Li, M., Li, E., et al. (2023), Pedal and Contrapedal Curves of Framed Immersions in the Euclidean 3-Space, Mediterr. J. Math., 20, 204
- [21] Yazıcı, B.D., Karakus, S.Ö., Tosun, M., (2021), On the classification of framed rectifying curves in Euclidean space, Math. Meth. Appl. Sci. 45(18), 1-10
- [22] Yazici, B.D., Karakus, S.Ö., Tosun, M., (2021), Framed normal curves in Euclidean space, Tbilisi Math. J., 27-37
- [23] Yılmaz, B., Has, A., (2019), New Approach to Slant Helix, International Electronic Journal of Geometry, 12(1), 111-115
- [24] Yu, H., Pei, D., Cui, X., (2015), Evolutes of fronts on Euclidean 2-sphere, J. Nonlinear Sci. Appl. 8, 678-68

## Exploring an Alternative Residual-based Control Chart for Monitoring Overdispersed Count Profile

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key-words: Profile monitoring, Conway-Maxwell-Poisson distribution, residual control chart.

#### Abstract:

The Conway-Maxwell-Poisson (COM-Poisson) profile is well-known as one of the effective methods for representing the relationship between one or more explanatory variables and a count response with varying dispersion. This study explores the application of a residual-based Shewhart control chart for monitoring the COM-Poisson profile with overdispersed response in the presence of multicollinearity. The monitoring procedure encompasses a model parameter estimation by utilizing variations of the *r*-*k* class estimator and the calculation of deviance residuals. It is followed by monitoring those deviance residuals via control charts to identify the deviations from the control state. A real-life data set that is adjusted to fit the assumptions is employed to illustrate the effectiveness of the approach allowing a comprehensive evaluation of the control chart performances within this context.

- [1] Conway R.W. and Maxwell W.L., (1962), A queuing model with state dependent service rates, Journal of industrial engineering, vol. 12, pp. 132-136
- [2] Kirbia B.M.G., (2022), More than hundred (100) estimators for estimating the shrinkage parameter in a linear and generalized linear ridge regression models, *Journal of Econometrics and Statistics, vol. 2, pp. 132-136*
- [3] Abbasi A. and Ozkale M.R. (2021), The *r-k* class estimator in generalized linear models applicable with simulation and empirical study using a Poisson and Gamma responses, *Hacettepe Journal of Mathematics and Statistics*, vol. 50, pp. 594-611

## A New Generalization of Stancu-Schurer Operators

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key-words: Stancu-Schurer operators, modulus of continuity, Voronovskaja type theorem, Grüss-Voronovskaja type theorem.

#### Abstract:

In the present study, we introduce a new generalization of Stancu-Schurer operators and study their approximation properties. We obtain a uniform approximation result using the well-known Korovkin theorem and the rate of convergence in terms of the modulus of continuity. Then, we prove the Voronovskaja and Grüss-Voronovskaja type theorems. Also, we give some numerical examples.

- [1] Barbosu D., (2003), Schurer-Stancu type operators, Studia Universitatis Babeş-Bolyai Mathematica, XLVIII (3): 31-35.
- [2] Cetin N., and Acu A. M., (2021), Approximation by  $\alpha$ -Bernstein-Schurer-Stancu Operators, Journal of Mathematical Inequalities, 15(2).
- [3] Gal S., and Gonska, H., (2015), Grüss and Grüss-Voronovskaya-type estimates for some Bernstein-type polynomials of real and complex variables, *Jaen Journal on Approximation*, 7(1): 97–122.
- [4] Schurer F., (1962), Linear positive operators in approximation theory, Math. Inst. Techn. Univ. Delft Report.
- [5] Stancu D.D., (1983), Approximation of functions by means of a new generalized Bernstein operator, *Calcolo,20: 211–229. doi: 10.1007/BF02575593*

## **Generalized Meromorphic Functions**

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key-words: Generalized meromorphic functions, boundary value problems, iterative transformation.

#### Abstract:

The generalized solutions of a special class of regular Carleman-Vekua equations (generalized meromorphic functions) will be studied from the point of view of the theory of functions as well as from the point of view of the analysis of boundary value problems of the theory of functions; sufficiently important information about the behavior of the generalized meromorphic functions in the neighborhood of a point at infinity (and therefore the structure) is established; for the generalized meromorphic functions in some sense natural boundary problems are studied and their complete analysis is obtained. It should be also mentioned that the obtained results are new for the classical meromorphic functions as well. All the above results are essentially based on the iterative transformation of the meromorphic functions studied by the authors. The obtained results continue the research started in the monograph [1].

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## References

 Akhalaia G., Giorgadze G., Jikia V., Kaldani N., Makatsaria G., Manjavidze N., (2012), Elliptic Systems on Riemann Surfaces, Bulletin of TICMI, vol. 13, pp. 1-154

## **Odd-order Symmetric Operators on Compact Star Graphs**

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key-words: Quantum graphs, boundary value problems, symmetric operators, odd-order differential operators.

#### Abstract:

A quantum graph refers to a metric graph (a graph in which the edges are identified with the subintervals of the real line such that the endpoints of the intervals corresponding to the same vertex are identified) and a differential operator acting on it. Quantum graphs have been used as valuable tools for modeling physical phenomena that take place in a graph-like structure [1]. The most frequent differential operators considered on quantum graphs are second-order (Laplace or Schrödinger) operators [1]. The study of odd-order differential operators are significantly different from even-order counterparts [3, 4]. The only odd-order differential operator differential operator graphs is of first-order [2].

In this study, odd-order symmetric differential operators on compact quantum star graphs are considered. Differential operators acting on a compact quantum star graph that are generated by third and fifth-order formally symmetric differential expressions are constructed. The most general forms of non-local vertex conditions which yield symmetric third and fifth-order differential operators on compact star graphs are determined.

- [1] Berkolaiko, G. and Kuchment, P., 2013: Introduction to Quantum Graphs, American Mathematical Society.
- [2] Carlson, R., 1999, Inverse eigenvalue problems on directed graphs, Trans Am Math Soc, vol. 351, pp. 4069-4088.
- [3] Uğurlu, E., 2019, Regular third-order boundary value problems, Appl. Math. Comput., vol. 343, pp. 247-257.
- [4] Uğurlu, E., 2020, Regular fifth-order boundary value problems, Bull. Malays. Math. Sci. Soc., vol. 43, pp. 2105-2121.

## On the Mostar Index of Zero-Divisor Graphs Obtained from Commutative Rings

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key-words: Topological indices, Mostar index, distance in graphs, zero-divisor graph, commutative ring.

#### Abstract:

Topological indices have an important place in quantitative structure-property relationship (QSPR) studies. In chemical graph theory, a topological index is a fixed number associated with a graph under graph automorphisms. In 2008, Došlić et al.[1] introduced a new bond-additive topological index called the Mostar index, which serves as a measure of peripherality in graphs. The Mostar index of a graph  $\Gamma$  is defined as  $Mo(\Gamma) = \sum_{uv \in E(\Gamma)} |n_u(\Gamma) - n_v(\Gamma)|$ , where  $n_u(\Gamma)$  and  $n_v(\Gamma)$  denote the number

of vertices closer to vertex u than to vertex v, and the number of vertices closer to vertex v than to vertex u, respectively. The Mostar index is a measure that reveals how far a graph deviates from being distance-balanced. The aim of this paper is to give exact formulae for the Mostar index of the zero-divisor graph of the modulo integer ring for  $n = \lambda^k, \lambda\mu, \lambda^2\mu, \lambda^2\mu^2, \lambda\mu\rho$ , where  $\lambda, \mu$  and  $\rho$  are distinct prime numbers.

- [1] Došlić T., Martinjak I., Škrekovski R., Tipurić Spužević S. and Zubac I., (2018), Mostar index, Journal of Mathematical Chemistry, 56, 2995-3013
- [2] Hayat F. and Zhou B., (2019), On Mostar index of Trees with Parameters, Filomat, 33:19, 6453-6458
- [3] Akbar A. and Došlić T., (2021), Mostar index: Results and perspectives, Applied Mathematics and Computation, 404, 126245
- [4] Eğecioğlu Ö., Saygı E. and Saygı Z., (2021), The Mostar Index of Fibonacci and Lucas Cubes, Bulletin of the Malaysian Mathematical Sciences Society, 44, 3677-3687
- [5] Rehman M.U., Salman M., Khan S., Maden A.D. and Ali F., (2022), Mostar index of graphs associated to groups, Main Group Metal Chemistry, 45, 124-135
- [6] Gürsoy A., Gürsoy N.K. and Ülker A., (2022), Computing forgotten topological index of zero-divisor graphs of commutative rings, *Turkish Journal of Mathematics*, 46, 1845-1863
- [7] Asir T. and Rabikka V., (2022), The Wiener index of the zero-divisor graph of  $Z_n$ , Discrete Applied Mathematics, 319, 461-471

## Modeling Complex Social Systems: A Vision Using Networks

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key-words: General systems theory, labour markets, social system modelling.

#### Abstract:

If a complex social or economic system has a suitable model, then it becomes possible to utilize relevant mathematical tools, such as General Systems Theory (GST), to better understand the way the systems works. In this research, we apply these concepts to a social system's modelling, for a specific economic system. One benefit of the network perspective is that a large body of mathematics exists to help analyze many forms of network models. With a modeling of labor market through networks we obtain a better understanding on its overall functioning.

- [1] M. Lloret-Climent, J. A. Nescolarde-Selva, H. Mora-Mora, and M. T. Signes-Pont, (2018), A new network perspective in the study of labour markets, *Mathematical Methods in the Applied Sciences, vol. 41(6), pp. 2261-2268*
- [2] M. Lloret-Climent, J. A. Nescolarde-Selva, (2013), Data analysis using circular causality in networks, *Complexity, vol. 19(4)*, pp. 15-19
- [3] L. Bertalanffy, (1968): General System Theory: Foundations, Development, Applications, New York, NY, USA: Braziller

### **Graphs with Padovan Numbers as Degrees**

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key-words: Realizability, degree sequence, Padovan numbers, omega invariant.

#### Abstract

In this study, we consider the special number sequence called as Padovan numbers as the degree sequence of a graph. We give the necessary and sufficient conditions for the realizability of a given set of positive integers which consist of successive Padovan numbers and we call the graph a Padovan graph.

- [1] Aldous J.M., Wilson R.J., (2004): Graphs and Applications, An Introductory Approach, Springer, London
- [2] Delen S., Cangul I.N., (2019), Extremal Problems on Components and Loops in Graphs, Acta Mathematica Sinica, English Series, 35(2), 161-171
- [3] Delen S., Cangul I.N., (2018), A New Graph Invariant, Turkish Journal of Analysis and Number Theory, 6 (1), 30-33
- [4] Demirci M., Ozbek A., Akbayrak O., Cangul I.N., (2021), Lucas Graphs, Journal of Applied Mathematics and Computing, 65 (1-2), 93-106
- [5] Demirci M., Cangul I.N., (2020): Tribonacci Graphs, The ITM Web of Conferences, 34, 01002, Third ICAMNM
- [6] Duran O., Omur N., Koparal S., (2020), On sums with generalized harmonic, hyperharmonic and special numbers, *Miskolc Mathematical Notes*, 21(2), 791–803
- [7] Gertsch A., (1997), Generalized harmonic numbers, Proc. Royal Acad. Sci. Paris, Ser. I Math., 324 (1), 7-10
- [8] Koparal S., Omur N., Sudemen K. N., (2022), Some Identities for Derangement Numbers, *Miskolc Mathematical Notes*, 23, 773-785
- [9] Macmahon P. A., (1915-1916): Combinatorial Analysis, Cambridge University Press
- [10] Stewart I.,(1996), Tales of a Neglected Number, Scientific American
- [11] Yurttas Gunes A., Delen S., Demirci M., Cevik A.S., Cangul I.N., (2020), Fibonacci Graphs, Symmetry-Basel, 12, 1383

## **Spinor Quasi Equations in Euclidean 3-space**

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key-words: Curve, Frenet-Serret Frame, Quasi Frame, Spinor.

#### Abstract:

Spinors used in orthogonal group theory such as Lorentz groups, SO(3) in mathematics and physics were first described by Elie Cartan in 1913. The expression of mutually orthogonal unit vector triplets consisting of complex vector space elements as a single vector with two complex components is called spinor. In this study, by considering spinors, which are an element of  $C^2$  space corresponding to Frenet vectors of a curve in 3-dimensional Euclidean space; Spinor formulas were obtained for the quasi frame. By using the relationships between the Quasi frame and the Serret-Frenet frame, the relationships between the spinors corresponding to the curves are investigated.

- Cartan, E., (1913), Les groupes projectifs qui ne laissent invariante aucune multiplicité plane, Bulletin de la Société Mathématique de France, 41, 53-96.
- [2] Cartan, E., (1981): The theory of spinors, Hermann
- [3] Hacısalihoğlu, H.H., (1983), Diferensiyel Geometri, İnönü University, Faculty of Arts and Sciences Publications Mathematics, 2, 895.
- [4] Erişir, T., Kardağ, N. C., (2019), Spinor Representations of Involute Evolute Curves in E3, Fundamental Journal of Mathematics and Applications, 2(2), 148-155.
- [5] Erişir, T., Eren, K., (2021), Spinor Representation of Directional q-Frame, Preprint, DOI:10.13140/RG.2.2.35983.30889.
- [6] del Castillo, T., G.F., Barrales, G.S., (2004), Spinor formulation of the differential geometry of curves, *Revista Colombiana de Matematicas*, 38(1), 27-34.
- [7] Dede, M., Ekici, C. and Tozak, H., (2015), Directional Tubular Surfaces, International Journal of Algebra, 9, 527-535.

## Soliton Dynamics in Optical Metamaterials Having Cubic-quintic Nonlinearity with Third and Fourth Order Dispersions

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key-words: Optical metamaterials, analytical methods, soliton solutions.

#### Abstract:

Metamaterials can be defined as a new class of artificial materials that are not found in naturally occurring materials and exhibit exceptional properties [1]. They are used to control and change light, sound and many other physical phenomena. Metamaterials are used in many areas such as public safety, remote aviation, solar energy management, sensor identification, medical device production and so on.

Along them, optical metamaterials with negative refractive index have many applications such as super resolution imaging, invisibility cloak, efficient energy generation [2]. The aim of the study is to analyze optical metamaterial models and determine their properties and nonlinear pulse propagation, thus contributing to the manufacturing process. To this purpose, a nonlinear optical metamaterial model with cubic-quintic nonlinearity, detuning inter-modal dispersion along with nonlinear third and fourth order dispersion terms has been considered [3,4]. By means of two different methods, exact analytical solutions to the aforementioned model have been attained. Certain values of physical parameters have been chosen and optical wave patterns have been determined and simulated with graphs.

- [1] Valipour A., Kargozarfard M.H., Rakhshi M., Yaghootian A. and Sedighi, H.M., (2021), Metamaterials and their applications: An overview, *The Journal of Materials: Design and Applications*, vol. 236, pp. 2171-2210
- [2] Cai W. and Shalaev V., (2010): Optical Metamaterials, Springer
- [3] Hubert M.B., Nestor S., Betchewe G., Biswas A., Khan S., Doka S.Y., Zhou Q., Ekici M. and Belic M., (2019), Dispersive solitons in optical metamaterials having parabolic form of nonlinearity, *Optik, vol. 179, pp. 1009-1018*
- [4] Mathanaranjan T., Kumar D., Rezazadeh H. and Akinyemi L., (2022), Optical solitons in metamaterials with third and fourth order dispersions, *Optical and Quantum Electronics, vol. 54 (271), pp. 1-15*

On the Recursive Sequence  $x_{n+1} = \frac{x_{n-8}}{1+x_nx_{n-1}x_{n-2}x_{n-3}x_{n-4}x_{n-5}x_{n-6}x_{n-7}}$ 

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key-words: Difference equations, recursive sequences, equilibrium point.

#### Abstract:

Difference equations appear naturally as discrete analogs and as numerical solutions of differential and delay differential equations, having applications in biology, ecology, physics.

Difference equations are used in a variety of contexts, such as in economics to model the evolution through time of variables such as gross domestic product, the inflation rate, the exchange rate, etc. They are used in modeling such time series because values of these variables are only measured at discrete intervals. In econometric applications, linear difference equations are modeled with stochastic terms in the form of autoregressive (AR) models and in models such as vector autoregression (VAR) and autoregressive moving average (ARMA) models that combine AR with other features.

In this paper, we study positive solutions and attractivity of the rational difference formula

$$x_{n+1} = \frac{x_{n-8}}{1 + x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4} x_{n-5} x_{n-6} x_{n-7}}$$

where initial values are nonnegative real numbers.

- [1] Ahmed A.M., Samir A.M., and Aljoufi L.S., (2022), Expressions and dynamical behavior of solutions of a class of rational difference equations of fifteenth-order, J. Math. Computer Sci., 25, pp. 10-22.
- [2] Elsayed E.M., Alharthi M.T., (2023), Qualitative Behavior and Solutions of Sixth Rational Difference Equations, Pure and Applicable Analysis 2023, pp. 1-26.
- [3] Karataş R., Gelişken A., A Solution Form of A Higher Order Difference Equation, Konuralp Journal of Mathematics, 9(2), pp. 316-323.
- [4] Simsek D., Cinar C., and Yalcinkaya I. (2008), On the recursive sequence  $x_{n+1} = x_{n-(5k+9)}/1 + x_{n-4}x_{n-9} \dots x_{n-(5k+4)}$ , Taiwanese Journal of Mathematics, 12(5), pp. 1087-1099.
- [5] Elabbasy, E.M., El-Metwally, H., and Elsayed, E.M. (2006), On the Difference equation xn + = axn bxn/(cxx dxn 1). Advances in Difference Equations, 2006, pp. 1-10.

## Algebra of Fibred Relations

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key-words: Finite set, fibred correspondence, relations, primitive idempotents.

#### Abstract:

Let k be a field, X a finite set and A be an abelian group. The k-algebra of all relations on X has been an object of study for a long time, as can be seen from the papers [1], [2], [3]. Recently, it has been receiving a modern functorial treatment in a series of papers [4], [5], [6], [7] by Bouc and Thevenaz. The multiplication in this algebra is given by the \*-product and it also appears in the category of bisets where instead of finite sets the finite groups are considered. By equipping each group with one dimensional A-characters one obtains the fibred biset category and thanks to the extra structure that comes with the finite groups, their multiplication can be defined without complications. However, although one can also equip the finite sets with characters, the canonical choice for the multiplication is not as clear as the previous case due to the lack of a group structure. We shall provide a solution to this problem by defining the composition of fibred relations. We will also describe the algebra formed by a set and its characters by parametrizing certain primitive idempotents.

## References

- [1] Schwarz S., (1970), On the semigroup of binary relations on a finite set Czechoslovak Mathematical Journal, 20(95), 632-679
- [2] Bremner M.R., El Bachraoui M., (2010), On the semigroup algebra of binary relations *Communications in Algebra, 38,* 3499-3505
- [3] Kim K.H., Roush F., (2020): Linear Representations of Semigroups of Boolean Matrices *Proceedings of American Mathematical Society*, 63, 203-207
- [4] Bouc S., Thévenaz J., (2018), Correspondence functors and finiteness conditions, Journal of Algebra, 495, 150-198
- [5] Bouc. S., Thévenaz J., (2019), Correspondence functors and lattices Journal of Algebra, 518, 453-518
- [6] Bouc. S., Thévenaz J., (2020), Tensor product of correspondence functors Journal of Algebra, 518, 453-518
- [7] Bouc. S., Thévenaz J., (2020), The algebra of Boolean matrices, correspondence functors, and simplicity *Journal of Combinatorial Algebra*, 4(3), 215-267

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## Some Tauberian Conditions for the Logarithmic Integrability

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key-words: Tauberian theorem, slowly oscillating function, summability method of integrals.

#### Abstract:

In this study, we give the concept of slow oscillation with respect to logarithmic summability method and general logarithmic control module. Our purpose is to prove some Tauberian theorems for the logarithmic integrability method by using these concepts.

- [1] Móricz, F., (2013), Necessary and sufficient Tauberian conditions for the logarithmic summability of functions and sequences, *Studia Math.*, 219, 109–121.
- [2] Totur, Ü., Okur, M. A., (2018), On Tauberian conditions for the logarithmic methods of integrability, Bull. Malays. Math. Sci. Soc., 41, 879–892.
- [3] Okur, M. A., Totur, Ü., (2019), Tauberian theorems for the logarithmic summability methods of integrals, Positivity, 23, 55-73.
- [4] Ishiguro, K., (1962), On the summability methods of logarithmic type, Proc. Japan Acad., 38, 703–705.
- [5] Ishiguro, K., (1963), A converse theorem on the summability methods, Proc. Japan Acad., 39, 38-41.
- [6] Sezer, S.A., Çanak, İ., (2018), Tauberian Theorems for the Summability Methods of Logarithmic Type, Bull. Malays. Math. Sci. Soc., 41, 1977–1994.
- [7] Sezer, S.A., (2020), Logarithmic means of sequences of fuzzy numbers and a Tauberian theorem, Soft Computing., 24, 367–374.
- [8] Sezer, S.A., Çanak, İ., (2020), Tauberian Conditions of Slowly Decreasing Type for the Logarithmic Power Series Method, Proc. Natl. Acad. Sci. India Sect. A Phys. Sci., 90, 135–139.

## Minimal Generators of Syzygy Modules and Colon Ideals Via Matrices

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key-words: Smith normal form, Cauchy-Binet Formula, modules.

#### Abstract:

Let  $R = \mathbf{K}[x]$  be a univariate polynomial ring over an algebraically closed field  $\mathbf{K}$  of characteristic zero. Let  $A \in M_{m,m}(R)$  be an  $m \times m$  matrix over R whose determinate  $\det(A) = d(x)$  is irreducible in R. Utilizing linear-algebraic methods, this paper investigates relationships between minimal sets of generators: first the colon ideal (J : I) where  $I = \langle f_1, \ldots, f_m \rangle$  and  $J = \langle g_1, \ldots, g_m \rangle$  with  $[g_1, \ldots, g_m] = [f_1, \ldots, f_m]A$  is described; then the "Koszul-like" generators of a syzygy submodule are connected to a basis for the syzygy module of  $f_1, \ldots, f_m$ ; finally, a basis for the syzygy module of  $f_1, \ldots, f_m$  is linked to a basis for the syzygy module of  $g_1, \ldots, g_m$ .

- [1] Adkins W., Weintraub S., (1992): Algebra, Graduate Texts in Mathematics 136, Springer-Verlag
- [2] Bose N.K., (1995): Multidimensional Systems Theory and Applications, Springer
- [3] Cox D., Little J. and O'shea D., (2005): Using Algebraic Geometry, Second Edition Graduate Texts in Mathematics, 185, Springer
- [4] Dobbs D.E., (1997): On minimal generating sets of modules over a special principal ideal ring, *Lecture Notes in Pure and Appl. Math, Dekker, New York, 185, 241-250*
- [5] Eisenbud D., (1995): Commutative algebra. With a view toward algebraic geometry, Graduate Texts in Mathematics, 150, Springer-Verlag, New York
- [6] Gué D.T., (1985), Minimum number of generators of the lattice of submodules of a semisimple module, *Journal of Soviet Mathematics*, 30, 1872-1874
- [7] Kravchenko A.A., (1984), On the minimum number of generators of the lattice of subspaces of a finite dimensional linear space over a finite field, *Journal of Soviet Mathematics*, 27(4)
- [8] Wiesinger-Widi M., (2011), Gröbner Bases and Generalized Sylvester Matrices, *ACM Communications in Computer Algebra*, 45(2), 176, 137-138

## On a Generalization of $\bigoplus_{\delta_{ss}}$ -Supplemented Modules

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key-words: Co-coatomic submodule, left  $\delta_{ss}$ -perfect ring,  $\oplus_{\delta_{ss}}$ -co-coatomically supplemented modules.

#### Abstract:

In this study, we describe the notion of  $\bigoplus_{\delta_{ss}}$ -co-coatomically supplemented modules as a proper generalization of  $\bigoplus_{\delta_{ss}}$ -supplemented modules. Under some conditions, we provide various algebraic properties of these modules. In particular, we show that a ring R is left  $\delta_{ss}$ -perfect if and only if RR is  $\bigoplus_{\delta_{ss}}$ -co-coatomically supplemented.

- [1] Alizade R., Güngör S., (2018), ⊕-Co-coatomically supplemented and co-coatomically semiperfect modules, *Hacettepe J. Math. Stat.*, 47(6), 1417-1426
- [2] Eryılmaz F., Öztürk Sözen E., (2023), On a generalization of ⊕-co-coatomically supplemented modules, *Honam Math. J.*, 45(1), 146-159
- [3] Öztürk Sözen E., (2022), On  $\oplus_{\delta_{ss}}$ -supplemented modules, Publ. de l'Institut Math., Nouvelle série, tome, 112(126), 59-69

# $(\alpha, \beta)$ - Contractive Mapping in Intuitionistic Fuzzy Metric-Like Spaces and Fixed Points

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key-words: 1 - M - complete intuitonistic fuzzy metric-like spaces, intuitionistic fuzzy ( $\alpha, \beta$ )-contractive mapping, fixed point.

#### Abstract:

In this study, we modify the concepts of Cauchy sequence and completeness in intuitionistic fuzzy metric-like spaces [4] by defining the notions of 1– Cauchy sequence and 1–complete intuitionistic fuzzy metric like spaces. Using the definition of intuitionistic fuzzy ( $\alpha$ ,  $\beta$ )-contraction mapping, we extend the fixed point results to intuitionistic fuzzy metric-like spaces. Furthermore, we present several examples to show the significance of our results.

- [1] J. Jeyachristy Priskillal, P. Thangavelu, (2017), Intuitionistic Fuzzy ( $\psi$ ,  $\eta$ )-Contractive Mapping and Fixed Points, *Journal of Analysis and Number Theory, vol.5 (2), pp. 105-108.*
- [2] Satish Shukla, Mujahid Abbas (2014), Fixed point results in fuzzy metric-like spaces, *Iranian Journal of Fuzzy Systems*, vol.11 (5), pp. 81-92.
- [3] Satish Shukla, Dhananjay Gopal, Antonio-Francisco Roldán-López-de-Hierro, (2016), Some fixed point theorems in 1 *M*-complete fuzzy metric-like spaces, *International Journal of General Systems*, vol.45 (7-8), pp. 815-829.
- [4] Şuara Onbaşıoğlu, Banu Pazar Varol, (2023), Intuitionistic Fuzzy Metric-like Spaces and Fixed-Point Results, *Mathematics*, vol.11 (1902), pp. 1-15.

## A Class of Lcd Codes Based on Circulant Matrices

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key-words: Linear codes, LCD codes, circulant matrices.

#### Abstract:

A linear code C is called a linear code with complementary dual (LCD) if it satisfies that  $C \cap C^{\perp} = 0$ . A square matrix in which each row (after the first row) has the elements of the previous row shifted cyclically one place right, is called a circulant matrix. In this study, we give a class of LCD codes based on the invertible circulant matrices.

- [1] Massey J.L., (1992), Linear codes with complementary duals, Discrete Mathematics, 106, 337-342
- [2] Gray, R.M., (2006), Toeplitz and circulant matrices: A review, Foundations and Trends in Communications and Information Theory, 2(3), 155-239
- [3] Siap I., Akin H. and Koroglu M.E., (2013), The Reversibility of (2r + 1)-cyclic rule cellular automata, TWMS J. Pure Appl. Math., 4, 215-225

# Hermite-Hadamard Type Inequalities for Multiplicatively (s, P)-Functions

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key-words: Hermite-Hadamard inequalities, integral inequalities, multiplicative integral, Non-Newtonian calculus.

#### Abstract:

In this paper, the concept of multiplicatively (s, P)-function is introduced and studied. Hermite-Hadamard integral inequalities are established for this newly class of functions. Also, some inequalities of Hermite-Hadamard type for the product and quotient of multiplicatively (s, P)-functions are obtained. Furthermore, novel inequalities involving multiplicative integrals for product and quotient of positive convex and multiplicatively (s, P)-functions. Some applications to special means are also given.

# Simpson, Midpoint and Trapezoid Type Inequalities for Multiplicatively s-Convex Functions

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key-words: Simpson's inequality, integral inequalities, multiplicative integral, Non-Newtonian calculus.

### Abstract:

In this study, we first prove some new identities for multiplicative differentiable functions. Based on this identity, we establish new inequalities of Simpson, midpoint and trapezoid type for multiplicatively *s*-convex functions. Some applications to special means are also given.

(2023)

## **Physically and Biological Interpretations of the Elliptical Quaternions**

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key-words: Kinematics, special curves and surfaces, quaternion algebra, frame fields, accretive surface growth, magnetic flux surfaces.

#### Abstract:

Elliptical rotation is an act of a point on an ellipse through some angle about an axis. A unit elliptic quaternion generates elliptical rotation on the ellipsoid. In the present paper, using the elliptical motion, we generate elliptical canal surfaces (ECSs). This form of (ECSs, not only surfaces with elliptical cross-sections but also surfaces that elliptical motion is formed during their formation. Namely, any point on the cross-section curve of the surface traces an elliptical trajectory during the formation of the surface. Thus, this form of surface is more compatible with the natural surface formation model. Since the geometric structure that can best explain this formation model is the elliptic quaternion algebra, we generate a variety of ECSs through elliptical quaternions. To generate ECSs, we use the elliptical rotation matrix corresponding to the unit elliptic quaternion q and also construct the ECSs via homothetic motion in scalar 3-space  $\mathbb{R}^3_{a_1,a_2,a_3}$ . As an application, we give two special forms of ECSs. The first model is the accretive surface growth model with an elliptical cross-section and elliptical cell tract. The second model is we obtain elliptical magnetic flux tube surfaces (EMFTs) with elliptical cross-section and elliptical formation. The most important feature of these surface formations is that they are produced using elliptic quaternion algebra, which makes operations and MAPLE drawings very easy. As a result, surfaces that are closer to natural formations, physically meaningful, and pleasing to the eye have been produced. In addition, we provide various biological and physical examples and visualize the images of these surfaces by using the MAPLE program

- [1] Hans M., (2009): The Algorithmic Beauty of Sea Shells, Springer
- [2] Illert C., (1989), Formulation and solution of the classical problem, II Tubular three dimensional sur- faces, *Nuovo Cimento*, *11*, 761-780
- [3] Moseley H., (1838), On the Geometrical Forms of Turbinated and Discoid Shells, Phil. Trans. R. Soc. Lond., 128, 351-370
- [4] Özdemir M., (2016), An Alternative Approach to Elliptical Motion, Adv. Appl. Clifford Algebras, 26. 279-304
- [5] Özdemir Z., (2021), A geometrical and physical interpretation of quaternionic generalized magnetic flux tubes, *Chaos, Solitons and Fractals*, 143, 110541
# A High-Dimensional Categorical Perspective on 2-Crossed Modules

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key-words: 2-crossed module, tricategory, homotopy.

#### Abstract:

In this study, we will express the 2-crossed module of groups from a high-dimensional categorical perspective. According to simplicial homotopy theory, a 2-crossed module is the Moore complex of a 2-truncated simplicial group [2]. Therefore, 2-crossed module is an algebraic homotopy model for the homotopy 3-types. For homotopy modeling, see [5, 8].

Tricategories are a 3-dimensional generalization of the bicategory concept [4, 6]. Any tricategory is triequivalent to Gray-category, where Gray is a category enriched over the monoidal category 2Cat equipped with the Gray tensor product [4]. Briefly, a Gray-category is a semi-strict 3-category for homotopy 3-types [3, 1]. Naturally, the tricategory perspective is used in homotopy theory. In [7, 9], the 2-crossed module is associated with the concept of Gray-category. Our aim in this study is to obtain a single-object tricategory from any 2-crossed module of groups.

- [1] Crans S. E., (1999), A tensor product for Gray-categories, Theory and Applications of Categories, 5(2), 12-69
- [2] Conduché D., (1984), Modules croisés généralisés de longueur 2, Journal of Pure and Applied Algebra, 34(2-3), 155-178
- [3] Gray J. W., (2006): Formal category theory: adjointness for 2-categories, Springer
- [4] Gordon R., Power A. J. and Street R., (1995): Coherence for tricategories, American Mathematical Society
- [5] Gohla B. and Martins J. F., (2013), Pointed homotopy and pointed lax homotopy of 2-crossed module maps, Advances in Mathematics, 248, 986-1049
- [6] Gurski N., (2013): Coherence in three-dimensional category theory, Cambridge University Press
- [7] Kamps K. H., Porter T., (2002), 2-groupoid enrichments in homotopy theory and algebra, K-theory, 25(4), 373-409
- [8] Martins J. F., (2011), The fundamental 2-crossed complex of a reduced CW-complex, *Homology, Homotopy and Applications*, 13(2), 129-157
- [9] Martins J. F., Picken R., (2011), The fundamental Gray 3-groupoid of a smooth manifold and local 3-dimensional holonomy based on a 2-crossed module, *Differential Geometry and its Applications*, 29(2), 179-206

## Convergence of a Family of Bivariate Sampling-Durrmeyer Operators in Weighted Spaces of Functions

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key-words: Bivariate sampling Durrmeyer operators, pointwise and uniform convergence, Voronovskaja theorem, weigted spaces of functions

#### Abstract:

In this talk, we give the basic concept of sampling type approach methods and aproximation properties of bivariate sampling Durrmeyer operators for functions belonging to weighted space of functions. We first present well-definiteness of bivariate sampling Durrmeyer operators in weighted spaces of functions, pointwise and uniform convergence of these series. Next, we determine rate of convergence via bivariate weighted modulus of continuity. Moreover a quantitative Voronovskaja theorem is also present to obtain a rate of pointwise convergence and an upper estimate for this convergence.

- [1] Acar T., Aral A. and Rasa I., (2016), The new forms of Voronovskaya's theorem in weighted spaces, *Positivity, vol. 20(2), pp.* 25-40
- [2] Acar T. and Turgay M., (2023), Approximation by bivariate generalized sampling series in weighted spaces of functions, *Dolomites Res. Notes Approx.*, vol. 16, pp. 11-22
- [3] Alagöz O., Turgay M., Acar T. and Parlak M., (2022), Approximation by Sampling Durrmeyer Operators in Weighted Space of Functions, *Numer. Funct. Anal. Optim., vol. 43(10), pp. 1223-1239*
- [4] Bardaro C., Butzer P. L., Stens R. L. and Vinti G., (2007), Kantorovich-type generalized sampling series in the setting of Orlicz space, Sampl. Theory Signal Image Process, vol. 6(1), pp. 29-52
- [5] Bardaro C. and Mantellini I., (2014), Asymptotic expansion of generalized Durrmeyer sampling type series, *Jaen J. Approx.*, vol. 6(2), pp. 143-165
- [6] Costarelli D., Piconi M. and Vinti G., (2023), On the convergence properties of sampling Durrmeyer-type operators in Orlicz spaces, *Mathematische Nachrichten, vol. 296 (2), pp. 588-609*
- [7] Ispir N., (2001), On modified Baskakov operators on weighted spaces, Turkish J. Math., vol. 26(3), pp. 355-365

## Logarithmic Coefficients for Starlike Functions Associated with Generalized Telephone Numbers

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key-words: Univalent functions, logarithmic coefficients, starlike functions, telephone numbers.

#### Abstract:

The objective of the present paper is to study the logarithmic coefficients of the class  $S_T^*(\mu)$  of starlike functions which is related with generalized telephone numbers, by using bounds on some coefficient functional for the family of functions with positive real part.

- Ali, R. M., Chandrashekar, R. and Ravichandran, V. A., 2011, Janowski starlikeness for a class of analytic functions, *Applied Mathematics Letters*, vol. 24(4), pp. 501-505.
- [2] Deniz, E., 2022, Sharp coefficients bounds for starlike functions associated with generalized telephone numbers, *Bull. Malays. Math. Sci. Soc., vol. 44(3), pp. 1525-1542.*
- [3] Duren, P. L., 1983: Univalent Functions, Springer-Verlag, New York.
- [4] Girela, D., 2000, Logarithmic coefficients of univalent functions, Ann. Acad. Sci. Fenn. Math. vol. 25, pp. 337-350.

## New Fixed Point Results in Weak Partial Metric Spaces

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key-words: Fixed point, weak partial metric space.

#### Abstract:

The concept of the partial metric p on a non-empty set X was introduced by Matthews [3] and redefined by Heckmann [2] as the weak partial metric with minor modifications. The most remerkable feature here is that, unlike the classical metric, the distance from the point itself may not be zero. In [1], the authors examined new fixed point results in weak partial metric spaces. Our aim in this talk is to examine new fixed point problems in weak partial metric spaces.

- [1] Durmaz, G., Acar, Ö., Altun, I., (2013), Some fixed point results on weak partial metric spaces, Filomat, vol. 27(2), pp. 317-326
- [2] Heckmann, R., (1999), Newblock Approximation of metric spaces by partial metric spaces, *Appl. Categ. Structures, vol. 7, pp.* 71-83
- [3] Matthews S. G., (1994), Partial metric topology, Proc. 8th Summer Conference on General Topology and Applications, *New York Acad. Sci.*, vol. 728, pp. 183-197.

## Some Fixed Point Results on Ultrametric Spaces Endowed with Graph

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key-words: Dynamic programming, Fixed Point, F-contraction, Functional equation, Ultrametric space.

#### Abstract:

In this talk, deals with new fixed point theorems through G-strongly contractive maps. The results are confered in spherically complete ultrametric space, which is invested with a graph, for single valued mappings. The specific cases of the results which produce the existing ones are presented and some examples are contributed to illustrating our outcomes. Also, an application is offered that is implemented in dynamic programming.

## Normal Automorphisms on Leibniz Algebras

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key-words: Leibniz algebra, normal automorphism, inner automorphism.

#### Abstract:

Let L be a free metabelian Leibniz algebra with generating set  $X = \{x_1, ..., x_n\}$  over the field K of characteristic 0. An automorphism  $\theta$  of L is said normal automorphism if each ideal of L is invariant under  $\theta$ . In this study, it is proven that every normal automorphism of L is an IA-automorphism and every normal automorphism is inner.

## References

[1] Bloh A., (1965), On a generalization of Lie algebra, Math. in USSR Doklady, 165(3), 471-473

[2] Drensky V., Cattaneo G.M.P., (2002), Varieties of Metabelian Leibniz Algebras, Journal of Algebra and Its Appl., 1(1), 31-50

[3] Endimioni G., (2010), Normal automorphisms of a free metabelian nilpotent group, Glasgow Math.J., 52, 169-177.

[4] Findik S., (2010), Normal and normally outer automorphisms of free metabelian nilpotent Lie algebras, Serdica Math. J., 36, 171-210.

[5] Loday J.L., (1993), "Une version non commutative des alg'ebres de Lie: les alg'ebres de Leibniz," Enseign. Math., 39, 269-293.

[6] Loday J.L., Pirashvili T., (1993), Universal Enveloping Algebras of Leibniz algebras and (co)Homology, Math. Ann., 296, 139-158.

[7] Lubotzky A., (1980), Normal automorphisms of free groups, J. Algebra, 63, 494.

[8] Oguslu N.S., (2020), Normal automorphisms of the metabelian product of free abelian Lie algebra, Algebra and Discrete Mathematics, 30(2), 230-234

[9] Romanovskii N.S., (1997), Normal automorphisms of free solvable pro-p-groups, Algebra Log., 36, 257.

[10] Romankov V.A., (1983), Normal automorphisms of discrete groups, Siberian Math. J., 24(4), 604-614.

[11] Tas Adiyaman T., Ozkurt Z., (2019), Automorphisms of free metabelian Leibniz algebras of rank three, Turk. J. Math., 43(5), 2262-2274.

[12] Tas Adiyaman T., Ozkurt Z., (2021), Automorphisms of free metabelian Leibniz algebras. Comm. Algebra, 49(10), 4348-4359.

[13] Timoshenko E.I., (2015), Normal automorphisms of a soluble product of abelian groups, Siberian Math. J., 56(1), 191-198.

## **Power Graphs of Some Graph Classes**

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key-words: Graph, power graph, omega invariant, path, cycle, star, complete graphs.

#### Abstract:

Path, cycle, star and complete graphs are the classical graph classes that are used in many combinatorial calculations made by graphs. The k-th power graph of a graph is obtained from the graph by adding a new edge between all pairs of vertices at maximum distance k. Omega invariant is a new topological and combinatorial graph invariant recently defined by Delen and Cangul. In this study, we concentrate on the k-th power graphs of the path, cycle, star and complete graphs and obtain several properties of them. Especially, the omega invariant and the number of edges of these power graphs are calculated. The results given here have many combinatorial and topological properties and applications.

- [1] Delen S., Cangul I.N., (2019), Extremal Problems on Components and Loops in Graphs, Acta Mathematica Sinica, English Series, 35(2), 161-171
- [2] Delen S., Cangul I.N., (2018), A New Graph Invariant, Turkish Journal of Analysis and Number Theory, 6(1), 30-33

## On the Mellin-Gauss-Weierstrass Operators Preserving Logarithmic Functions

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key-words: Mellin convolution operators, weighted approximation, Korovkin type theorem.

#### Abstract:

In this talk, we give a new generalization of Mellin-Gauss-Weierstrass operators. We calculate moments of the new operators and express the behaviour of the modified operators in some weighted spaces. Later, the weighted approximation properties of operators using weighted logarithmic modulus of continuity are stated.

- [1] Angeloni L., Vinti G., (2014), Convergence and rate of approximation in BV  $\phi$  (RN) for a class of Mellin integral operators, *Rend. Lincei-Mat. Appl.*, 25(3), 217-232.
- [2] Angeloni L., Vinti G., (2015), Approximation in variation for Mellin integral operators, PAMM, 15(1), 649-650.
- [3] Aral A., Acar T. and Kursun S., (2022), Generalized Kantorovich forms of exponential sampling series, *Anal. Math. Phys.*, 12 (2), 1-19.
- [4] Bardaro C., Mantellini I., (2011), Approximation properties for linear combinations of moment type operators, *Comput. Math. Appl.*, *62*(*5*), *2304-2313*.
- [5] Bardaro C., Mantellini I., (2011), Asymptotic behaviour of Mellin–Fejer convolution operators, *East J. Approx.*, 17(2), 181-201.
- [6] Bardaro C., Mantellini I., (2014), On Mellin convolution operators: a direct approach to the asymptotic formulae, *Integral Transf. Spec. Funct.*, 25(3), 182-195.
- [7] Bardaro C., Mantellini I., (2012), On the iterates of Mellin-Fejer convolution operators, Acta Appl. Math., 121(1), 213-229.
- [8] Bardaro C., Mantellini I., (2007), Voronovskaya-type estimates for Mellin convolution operators, Res. Math., 1(50), 1-16.
- [9] Butzer P.L., Jansche S., (1997), A direct approach to the Mellin transform, J. Fourier Anal. Appl., 3(4), 325-376.
- [10] Gadziev A.D., (1976), Theorems of the type of P. P. Korovkin's Theorems, Mat Zametki, 20(5), 781-786.
- [11] Kolbe W., Nessel R.J.I., (1972), Saturation theory in connection with Mellin transform methods, *SIAM J. Math Anal., 3*, 246-262.
- [12] Korovkin P.P., (1959): Linear operators and approximation theory, Translated from the Russian ed., Russian Monographs and Texts on Advanced Mathematics and Physics, Vol. III. Gordon and Breach Publishers, Inc., New York, Hindustan Publishing Corp. (India), Delhi.

## Minimizing the Pressure on Filter Surfaces Using Optimal Control

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key-words: EPS, flux, mathematical model, model parameters, optimal control analysis, optimal solution.

#### Abstract:

The bacterial buildup causes an irreversible attachment known as EPS to develop on filter surfaces over time. This layer starts to narrow the pores, requiring more pressure to be applied to the filter to maintain the flux. Since high pressure reduces the filter's lifespan, our primary objective is to reduce this pressure by minimizing irreversible bacterial attachment. To attain this goal, we first introduce a simplified mathematical model defining the accumulation dynamics, then determine the model parameters numerically. Finally, we employ optimal control analysis to find optimal solutions.

- [1] Cogan N., Chellam S., (2014), Title, Journal of Membrane Science, vol. 469, pp. 410-417
- [2] Cogan N., Li J., Badireddy A. R., Chellam S. (2016), Title, Journal of Membrane Science, vol. 520, pp. 337-344
- [3] Cogan N., Ozturk D., Ishida K., Safarik J., Chellam S. (2022), Title, Journal of Membrane Science, vol. 286, 120294

## Solutions of Congruence Equations via An Imprimitive Action of Some Modular Subgroups

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key-words: Congruence equations, modular subgroup, imprimitive action.

#### Abstract:

In the paper, we first calculate indexes of some modular subgroups and, using these, we solve a congruence equation  $x^2 + a^2 \equiv 0 \pmod{p^\beta}$  without the Girard-Fermat Theorem by using a special imprimitive action introduced by the above modular subgroups where p is a prime of the form  $p \equiv 1 \pmod{4}$ , a is any integer coprime with p, and  $\beta$  is a natural number. Finally, we put our conjecture at the end of the paper.

- [1] Akbaş, M., 2001, On suborbital graphs for the modular group, Bull. London Math. Soc. 33, 647-652.
- [2] Akbaş, M. and Singerman, D., 1990, *The Normalizer of*  $\Gamma(N)$  *in*  $PSL(2, \mathbb{R})$ , Glasgow Math. J. **32** 317-327.
- [3] Biggs, N. L. and White A. T., 1979, Permutation groups and combinatorial structures, LMS Lect. Note Ser., CUP, Cambridge.
- [4] Heath-Brown, D. R., 1984, Fermat's two-squares theorem, Invariant, 3-5.
- [5] Jones, G. A., Singerman, D. and Wicks, K., 1991, *The modular group and generalized Farey graphs*, LMS Lect. Note Ser. 160, 316-338.
- [6] Newman, M., 1962, The structure of some subgroups of the modular group, Illinois J. Math. 6, 480-487.
- [7] Şengül, H., 2021, Simge ve kongrüans denklemler, PhD Thesis, Karadeniz Technical University, Trabzon.
- [8] Varouchas, Y., 1984, Une démonstration élémentaire du théorème des deux carrés, La Caverne, I.R.E.M. de Lorraine, France, Bulletin No.6, février, 31-39.
- [9] Williams, K. S., 1985, *Heath-Brown's elementary proof of the Girard-Fermat theorem*, Carleton Coordinates, Department of Mathematics and Statistics, Carleton University, Ottawa, Ontario, Canada, 4-5.
- [10] Zagier, D., 1990, A one-sentence proof that every prime  $p \equiv 1 \pmod{4}$  is a sum of two squares, American Mathematical Monthly, **97**, 144.

## **Some Fixed Point Theorems In F-Metric Spaces**

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key-words: Fixed point, Presic type contraction, F-metric space.

#### Abstract:

Fixed point theory studies have been a popular research area in recent years, both in metric spaces and in some generalized metric spaces. One of the generalized metric spaces defined in recently is F-metric spaces. Any metric space is an F-metric but the converse is not true generally. Therefore, F is an important class of the metric distance function.

Let X be a non-empty set and  $f: X \to X$  be a mapping. If f(x) = x for any  $x \in X$ , then x is fixed point of f. Preŝić introduced this definition as f(x, x, ..., x) = x for the mapping  $f: X^k \to X$ . Also proved a fixed point theorem for a mapping satisfying generalized contraction.

In this study, the Preŝić type contraction principle will be defined in *F*-metric spaces and fixed point theorems will be proved for this type contraction.

- [1] Jleli M. and Samet B., (2018) On a new generalization of metric spaces, J. Fixed Point Theory Appl., vol. 20(3), Article No. 128.
- [2] Presic S.B., (1965) Sur une classe d'in ' equations aux di ' fferences finies et sur la convergence de certaines suites, Publ. Inst. Math., vol. 5 (19), pp., 75–78.

## **Common Fixed Point Theorems In b-Metric Spaces**

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key-words: Fixed point, Hardy-Roger type contraction, b-metric space

#### Abstract:

The Banach contraction principle is one of the first and most important results of the fixed point theory. According to this principle proved by Banach in 1922; "(X, d) is a complete metric space and  $f : X \to X$  is a self mapping. If there is a constant  $0 \le k < 1$  such that the inequality  $d(fx, fy) \le k.d(x, y)$  is provided for each for each  $x, y \in X$ , then f has unique fixed point". Fixed point theory has many generalizations of this principle used in many sciences as well as mathematics.

In this study, the rational type Hardy Roger contraction principle was defined in b-metric spaces and common fixed point results were proved for Hardy Roger type contraction via  $\lambda$ -sequences.

## References

[1] Gaba Y.U., (2017), Metric type spaces and  $\lambda$ -sequences Quaestiones Mathematicae, vol.40 (1), pp. 49-55.

- [2] Czerwik, S., (1993): Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Ostra., vol.1 pp. 5-11.
- [3] Hardy G.E., Rogers T.D. (1973), A generalization of a fixed point theorem of Reich, Can. Math. Bull, vol. 16, pp. 201-206.

## **On Exchange Rings**

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key-words:  $(\mathcal{G})$ -idempotent element,  $(\mathcal{G})$ -exchange ring,  $(\mathcal{G})$ -clean ring.

#### Abstract:

Exchange ring have been characterized by the property that for all  $a \in R$ , there exist an idempotent  $e \in aR$  such that  $(1 - e) \in (1 - a)R$ . A unit-picker is a map  $\mathcal{G}$  that associates to every ring R a well-defined set  $\mathcal{G}(R)$  of central units in R such that  $\mathcal{G}(R)$  is invariant under isomorphism of rings.

In [5] An element q in a ring R is called a  $\mathcal{G}$ -idempotent (or *quasi-idempotent* [4]) if  $q^2 = gq$  for some  $g \in \mathcal{G}$  or, equivalently,  $g^{-1}q$  is an idempotent for some  $g \in \mathcal{G}$ . In this work we introduce the notation of  $\mathcal{G}$ -exchange ring via  $\mathcal{G}$ -idempotent and give some relation between  $\mathcal{G}$ -exchange ring and (strongly)- $\mathcal{G}$  clean ring.

- [1] Camillo V., Yu H. P., (1994), Exchange rings, units and idempotents, Comm. Algebra, 22, 4737-4749
- [2] Lee T. K., Zhou Y., (2008), A class of exchange rings, Glasgow Math. J., 50, 509-522
- [3] Nicholson W. K., (1977), Lifting idempotents and exchange rings, Trans. Amer. Math. Soc., 229, 269-278
- [4] Tang G., Su H. and Yuan P., (2021), Quasi-clean rings and strongly quasi-clean rings, *Commun. Contemp. Math.*, 25(2), 2150079
- [5] Tang G., Zhou Y., (2022), Nil G-clean rings and strongly nil G-clean rings, J. Algebra Appl., 21(4), 2250077

# **Involutions of Hybrid Numbers**

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key-words: Hybrid numbers, involutions, anti-involutions

#### Abstract:

Hybrid numbers are generalization of the complex, hyperbolic and dual numbers. An involution or anti-involution is a self-inverse linear mapping. In this paper we present involutions and anti-involutions of hybrid numbers.

- [1] Ozdemir M., (2018), Introduction to Hybrid Numbers, Adv. Appl. Clifford Algebras, 28, 1-32
- [2] Ella A. E., Sangwine S. J., (2007), Quaternion involutions and anti-involutions, Computers and Mathematics with Applications, 53, 137-143
- [3] Bekar M., Yayli Y., (2016), Involutions in semi-quaternions, JGSP, 41, 1-16
- [4] Bekar M., Yayli Y., (2013), Involutions of complexified quaternions and split quaternions, *Adv. Appl. Clifford Algebras*, 23, 283-299
- [5] Bekar M., Yayli Y., (2017), Involutions in split semi-quaternions, Math. Meth. Appl. Sci., 41, 4491-4505

## **Spectral Analysis of Discontinuous Fractional Boundary Value Problem**

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key-words: Fractional boundary value problem, eigenvalue, eigenfunction.

#### Abstract:

In this paper, we considered the eigenvalues and eigenfunctions of the fractional boundary value problems and the transmission conditions. First, we presented some theorems that give the spectral properties of the studied problem. Then, at the end of the study, we finished with an example to illustrate the given theorems.

- [1] Atkinson F. V., (1964): Discrete and Continuous Boundary Problems, Academic Press, New York/London
- [2] Klimek M., Agrawal O.P., (2013), Fractional Sturm-Liouville Problem, Computers and Mathematics with Applications, 66(2013), 795-812
- [3] Podlubny I., (1999): Fractional Differential Equations, Academic Press, San Diego

## On Power Series Strong and Statistical Convergence for Double Sequences via Orlicz Function

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key-words: Power series method, double sequences, Orlicz function.

#### Abstract:

The presentation aims to give the notions of strong  $P_p^2$ -convergence and  $P_p^2$ -statistical convergence for double sequences with respect to an Orlicz function. Based on these new notions of convergence, new double sequence spaces and some of their properties are given. Also, some inclusion relations are studied. Finally, it is remark that the results can be obtained for modulus function under proper choices.

## References

- [1] Demirci K., (1996), Strong A-summability and A-statistical convergence Indian Journal Pure and Applied Mathematics, vol.27, pp. 589-593.
- [2] Krasnoselskii M. A. and Rutisky Y. B., (1961), Convex Function and Orlicz Spaces, Groningen, Netherlands.
- [3] Pringsheim A., (1900), Zur theorie der zweifach unendlichen zahlenfolgen Math. Ann., vol. 53, pp. 289-321.
- [4] Şahin Bayram N., (2022), P -strong convergence with respect to an Orlicz function, Turk J. Math, vol. 46, pp. 832-838.
- [5] Ünver M. and Orhan C., (2019), Statistical convergence with respect to power series methods and applications to approximation theory, *Numerical Functional Analysis and Optimization*, vol. 40(5), pp. 533-547.
- [6] Yıldız, S., Demirci, K., & Dirik, F. (2022), Korovkin theory via Pp-statistical relative modular convergence for double sequences. *Rendiconti del Circolo Matematico di Palermo Series 2, pp. 1-17.*
- [7] Yurdakadim T. and Taş E., (2022), Effects of Fuzzy Setting in Korovkin Theory via Pp-Statistical Convergence Romanian Journal of Mathematics and Computer Science, vol. 12 (2), pp. 1-8.

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## Kantorovich Stancu Type Dunkl Generalization of Szàsz Operators Including Two-variable Hermite polynomials

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key-words: Dunkl exponential, Sàzsz operators, modulus of continuity, Lipschitz functions, Twovariable Hermite polynomials, Peetre's K-functional

#### Abstract:

The aim of the present paper to introduce a Kantrovich Stancu type Dunkl generalization of Szász operators including two-variable Hermite polynomials defined by Krech [14]. Then, we want to investigate approximation properties for these operators with the help of Korovkin theorem. Moreover, we give other approximation results via the class of Lipschitz functions, classical modulus of continuity, second modulus of continuity and Peetre's K-functional

- Bernstein, SN: Démonstration du théoréme de Weierstrass fond 'ee sur le calcul des probabilités. Commun. Soc. Math.Kharkow 2(13), 1-2 (1912).
- [2] Cekim, B., Aktas, R., Tasdelen, F., A Dunkl-Gamma Type Operator in Terms of Generalization of Two-Variable Hermite Polynomials, Indian J Pure Appl Math 53, 727–735 (2022).
- [3] Appell, P., Kampe de Feriet, J. Hypergeometriques et Hyperspheriques: Polynomes d'Hermite, Gauthier-Villars, Paris, 1926.
- [4] Szász, O: Generalization of S. Bernstein polynomials to the infinite interval. J. Res. Natl. Bur. Stand. 45, 239-245 (1950).
- [5] Gupta, V, Vasishtha, V, Gupta, MK: Rate of convergence of the Sz'asz-Kantorovich-Bezier operators for bounded variation functions. Publ. Inst. Math. (Belgr.) 72, 137-143 (2006)
- [6] Altın, A., Do gru, O. and Tasdelen, F., The generalization of Meyer-K" onig and Zeller operators by generating functions, J. Math. Anal. Appl., 312 (1) (2005), 181-194.
- [7] Doğru, O., Ozarslan, M.A. and Tasdelen, F., On positive operators involving a certain class of generating functions, Studia Sci. Math. Hungar., 41 (4) (2004), 415-429.
- [8] Krech, G., A note on some positive linear operators associated with the Hermite polynomials, Carpathian J. Math., 32 (1) (2016), 71–77.
- [9] Sucu, S., Icöz, G. and Varma, S., On some extensions of Szász operators including Boas-Buck type polynomials, Abstr. Appl. Anal., Vol. 2012, Article ID 680340, 15 pages.
- [10] Olgun, A., İnce, H. G. and Tasdelen, F., Kantrovich type generalization of Meyer-König and Zeller operators via generating functions, An. S, tiint, Univ. "Ovidius" Constanta Ser. Mat., 21 (3) (2013), 209–221.

# An Optimal Control Strategy to Reduce the Gadget Addiction

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key-words: Hamilton formalism, SEIRS model, Runge-Kutta.

#### Abstract:

Technology, which has developed rapidly in recent years, brings disadvantages along with its advantages. This technology, which is used in almost every aspect of our lives, especially affects children and young people and causes them to become addicted to gadgets. For this reason, it can significantly affect both the life and education of individuals and cause serious health problems. In this study, a mathematical model representing this problem is discussed. An optimal treatment control is adapted to the model for individuals who can overcome such a problem, that is, individuals who have reached the infected level. For this purpose, the basic concepts of optimal control theory are utilized and Hamilton's formalism is used. Runge-Kutta 14 algorithm is used to obtain the numerical solutions. Simulations reveal of the problem and numerical results reveal how effective treatment control is on infected individuals. As a result, it is observed that treatment significantly affects infected individuals and even individuals in other compartments.

- [1] Suratna, Ayu Annisa and Cipta, Hendra and Sari, Rina Filia, (2023), Analisis Model Seirs terhadap Kecanduan Gadget Anak Usia Dini dengan Metode Runge-Kutta Orde-5, *MAJAMATH: Jurnal Matematika dan Pendidikan Matematika*, vol. 6(1), pp. 13–22
- [2] Lenhart S. and Workman J.T., (2007): Optimal control applied to biological models, CRC press
- [3] Fatahillah, Arif and Istiqomah, Maulida and Dafik, Dafik, (2021), Pemodelan Matematika Pada Kasus Kecanduan Game Online Menggunakan Metode Runge-Kutta Orde 14, *Limits: Journal of Mathematics and Its Applications, vol. 18(2), pp. 129–141*

## Statistical Inference for the Geometric Process with Skew-normal Distribution

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key-words: Geometric process, interval estimation, Monte Carlo simulation, point estimation.

#### Abstract:

In this study, we focus on statistical inference for the geometric process when the distribution of the first event is assumed to follow a skew-normal distribution. Several point estimation methods such as maximum likelihood, Anderson Darling, Cramér-von Mises and the least squares method are used to estimate the parameters of the geometric process. Interval estimation is also discussed in terms of maximum likelihood and bootstrap methods. A simulation study is performed to evaluate the accuracy of the proposed point and interval estimation methods. Finally, a numerical example is provided.

- [1] Biçer C., Biçer H. D., Kara M., and Aydoğdu H., (2019), Statistical inference for geometric process with the Rayleigh distribution, *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics*, 68(1), 149-160.
- [2] Demirci Biçer H., Biçer C., and Bakouch H. S. H., (2022), A geometric process with Hjorth marginal: Estimation, discrimination, and reliability data modeling, *Quality and Reliability Engineering International*, 38(5), 2795-2819.
- [3] Kara M., Güven G., Şenoğlu B., and Aydoğdu H., (2022), Estimation of the parameters of the gamma geometric process, *Journal of Statistical Computation and Simulation*, 92(12), 2525-2535.

#### On the Existence of Solutions for Multi-valued Operator Inclusions

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key-words: Multivalued operator, Hausdorff distance, generalized contractive, inclusion.

#### Abstract:

In this work, we present new existence theorems for the sum of two linear multi-valued operators under generalized contraction conditions. In particular, the obtained results are used to investigate the existence of solutions to some integral inclusions. Finally, we provide some applications that explain the results.

- [1] Lj. B. Ćirić, (2009), Multi-valued nonlinear contraction mappings., Nonlinear Anal., 71, 2716-2723.
- [2] S. B. Nadler, (1969), Multi-valued contraction mappings., Pacic J. Math. 30, 475-488.
- [3] X. Zhang, (2007), Common fixed point theorems for some new generalized contractive type mappings., *Journal of Mathematical Analysis and Applications*. 333, 780–786.
- [4] C.Temel, (2020), On some results of Krasnosel'skii's theorem for weak topology in Banach Space., *Fixed Point Theory.* 21, 309-318.

## A Graph Structure by means of Compliment Annihilator in Many-Valued Algebras

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key-words: Compliment annihilator,  $\triangle$ -connection graphs, many-valued algebras, graph folding.

#### Abstract:

This presentation unveils a groundbreaking concept known as  $\triangle$ -connection graphs, specifically designed for many-valued algebras, providing an innovative framework that is analogous to logical structures. These graphs are constructed by associating vertices with elements from many-valued algebras, and their connections are established through the utilization of the  $\triangle$ -connection operator.

In addition to introducing  $\triangle$ -connection graphs, this study introduces novel ideas in the form of graphs of equivalence classes. By incorporating the  $\triangle_{EQ}$ -connection operator and leveraging the complement of the annihilator within these algebraic structures, this concept introduces a fresh perspective on the categorization and organization of elements within many-valued algebras. Through extensive exploration, the research uncovers significant findings and essential insights into the characteristics and behavior of these graphs, anchored in fundamental algebraic properties.

Moreover, the investigation delves into the domain of graph folding within many-valued algebras, offering a comprehensive analysis of how graphs can be folded within these algebraic structures. By establishing a crucial link between the graph of equivalence classes and the graph folding of many-valued algebras, the study unravels an intricate relationship, further enriching our comprehension of the intersection between graph theory and many-valued algebraic structures.

To facilitate the practical implementation of these concepts, the research presents a collection of pioneering algorithms, meticulously crafted to address the unique challenges inherent in many-valued algebras. These innovative algorithms serve as invaluable tools for researchers and practitioners in various related fields, empowering them to navigate the complexities of many-valued algebraic systems and fostering advancements in these domains.

- [1] Gursoy A., Kircali Gursoy N., Oner T. and Senturk I., (2021), An alternative construction of graphs by associating with algorithmic approach on MV-algebras, *Soft Computing*, 25(21), 13201-13212
- [2] Božić I., Petrović Z., (2009), Zero-divisor graphs of matrices over commutative rings, *Communications in Algebra*, 37(4), 1186-1192
- [3] Chajda I., Halaš R. and Kühr J., (2009), Many-Valued quantum algebras, Algebra Univ., 60(1), 63-90
- [4] Gan A., Yang Y., (2015), The zero-divisor graphs of MV-algebras, Soft Computing, 28(4), 1829-1833
- [5] Cignoli R.L., d'Ottaviano I.M. and Munduci D., (2013): Algebraic foundations of many-valued reasoning, Springer Science and Business Media

## Point Estimation for the Inverse Rayleigh Distribution Under Type-II Left and Right Censoring

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key-words: Anderson-Darling statistic, Cramér-von Mises statistic, hyperbolic approximation, least squares, left and right censoring, modified maximum likelihood estimation.

#### Abstract:

The Inverse Rayleigh distribution is commonly used in reliability and survival analysis. In this study, the modified maximum likelihood estimate is obtained for the parameter of Inverse Rayleigh distribution under Type-II left and right censoring. Furthermore, maximum likelihood, Anderson-Darling, Cramér-von Mises type, and least squares estimators are also examined. The performance of the proposed estimators under different censoring schemes and parameter settings is evaluated by a Monte Carlo simulation. A numerical example is also provided.

- [1] Lalitha, S. and Mishra, A. (1996), Modified maximum likelihood estimation for Rayleigh distribution, *Communications in Statistics-Theory and Methods*, 25(2), 389-401
- [2] Ma, Y. and Gui, W. (2019), Pivotal inference for the inverse Rayleigh distribution based on general progressively Type-II censored samples, *Journal of Applied Statistics*, 46(5), 771-797

## Maximum Likelihood Estimates of Dependence Parameter in FGM Type Bivariate Gamma Distribution from Ranked Set Sampling

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key-words: FGM copula, ranked set sampling, dependence parameter, maximum likelihood estimation.

#### Abstract:

The Farlie-Gumbel-Morgenstern (FGM) distribution family was introduced by Morgenstern [1], Farlie [2] and Gumbel [3]. The FGM distribution family is quite attractive because of its simple structure and provide a modelling the dependence between two random variables. There are numerous studies in the literature because it is a widely used distribution. FGM type bivarite gamma distribution was defined by D'Este [4] and Gupta and Wong [5]. On the other hand, ranked set sampling (RSS) is a cost effective sampling method that is developed by McIntye [6]. First, Stokes [7] considered maximum likelihood (ML) estimator based on RSS for dependence parameter in bivariate normal distribution. Recently, ML estimators using ranked-based sampling designs were investigated by Sevil and Yildiz [8] for dependence parameter of Gumbel (type slowromancapi@) bivariate exponential distribution. The present work investigate ML estimators based on simple random sampling (SRS) and RSS for the dependence parameter of FGM type bivariate gamma distribution. Also, algorithms to generate sampling data from FGM type bivariate gamma distribution are provided.

- [1] Morgenstern D., (1956), Einfache Beispiele zweidimensionaler Verteilung, *Mitteislingsblatt für Mathematische Statistik*, 8, 234-235.
- [2] Farlie D.J., (1960), The performance of some correlation coefficients for a general bivariate distribution, *Biometrika*, 47, 307-323.
- [3] Gumbel E.J., (1960), Bivariate exponential distributions, Journal of the American Statistical Association, 55, 698-707.
- [4] D'Este G.A. (1981), A Morgenstern-type bivariate gamma distribution, Biometrika, 68, 339-340.
- [5] Gupta A.K. and Wong C. (1984), On a Morgenstern-type bivariate gamma distribution, Metrika, 31, 327-332.
- [6] McIntyre G.A. (1952), A method for unbiased selective sampling, using ranked set sampling, *Australian Journal of Agricultural Research*, *3*, 385-390.
- [7] Stokes S.L. (1980), Inference on the correlation coefficient in bivariate normal populations from ranked set samples, *Journal* of the American Statistical Association, 75, 989-995.
- [8] Sevil Y.C. and Yildiz T.O. (2022), Gumbel's bivariate exponential distribution: estimation of the association parameter using ranked set sampling, *Computational Statistics*, *37*, *1695-1726*.

## Some Bifurcations of a Fractional Order Discrete Prey-Predator Model

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key-words: Fractional order equation, flip bifurcation, Neimark-Sacker bifurcation.

#### Abstract:

This talk is dedicated to the investigation of a fractional order discrete predator-prey model. First of all the local stability of fixed points is studied. Then flip and Neimark-Sacker bifurcation at a positive fixed point is analyzed, respectively and some graphics are given to illustrate the results.

- [1] Barman, D., Roy, J., and Alam, S., (2022), Impact of wind in the dynamics of prey-predator interactions, *Mathematics and Computers in Simulation*, 191, 49-81.
- [2] Yousef, F., Semmar, B., and Al Nasr, K., (2022), Dynamics and simulations of discretized Caputo-conformable fractional-order Lotka–Volterra models, *Nonlinear Engineering*, 11(1), 100-111.
- [3] Baydemir, P., Merdan, H., Karaoglu, E., and Sucu, G., (2020), Complex dynamics of a discrete-time prey-predator system with Leslie type: stability, bifurcation analyses and chaos, *International Journal of Bifurcation and Chaos*, 30(10), 2050149.
- [4] Kuznetsov, Y. A., (1998): Elements of Applied Bifurcation Theory, Springer-Verlag, NY.

## On the Fuzzy Randic Energy and Fuzzy Randic Estrada Index of Fuzzy Graphs

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key-words: Fuzzy graphs, fuzzy randic index, fuzzy randic energy, fuzzy randic estrada index.

#### Abstract:

A fuzzy graph  $G = (V, \alpha, \beta)$  is a nonempty set V together with a pair of functions where  $\alpha : V \to [0, 1]$  is a fuzzy subset and  $\beta : V \times V \to [0, 1]$  is a fuzzy relation on  $\alpha$  such that  $\beta(u, v) \leq \min \{\alpha(u), \alpha(v)\}$  for all  $u, v \in V$ . The fuzzy Randic index is the half of sum  $[\alpha(u) \alpha(v) d(u) d(v)]^{\frac{1}{2}}$  where  $uv \in \beta$  and d(u) is the degree of vertex u, that is, d(u) is the sum  $\beta(u, v_i)$  which  $u \neq v_i$ . The corresponding fuzzy Randic matrix is the square matrix of order n whose (i, j)th entry whenever  $i \neq j$ , is  $[\alpha(u) \alpha(v) d(u) d(v)]^{\frac{1}{2}}$  and zero otherwise. In this study, fuzzy Randic energy and fuzzy Randic Estrada index are introduced and some bounds established for this energy and index.

- [1] Anjali N., Mathew S., (2013), Energy of a fuzzy graph, Annals Fuzzy Math. Inform., 6(3), 455-465
- [2] Gutman I., Trinajastic N., (1972), Graph theory and molecular orbitals. total 'phi-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, *17*(4), 535-538
- [3] Kalathian S., Ramalingam S., Raman S. and Srinivasan N., (2020), Some topological indices in fuzzy graphs, *Journal of Intelligent and Fuzzy Systems*, 39(5), 6033-6046
- [4] Randic M., (1975), Characterization of molecular branching, J. Amer. Chem. Soc., 97(23), 6609-6615
- [5] Rosenfeld A., (1975): Fuzzy graphs, Fuzzy sets and their applications to cognitive and decision processes, Elsevier, 77–95.
- [6] Zadeh L.A., (1965), Fuzzy sets, Information and control, 8(3), 338-353

## Mathematical Modeling of Detection, Spread and Control of COVID-19 Disease in presence of Illegal Migrants

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key-words: Illegal migrants, covid-19, stability analysis.

#### Abstract:

Around the world, there is concern over the COVID-19 illness. Numerous aspects of daily life are stopped by it. The COVID-19 epidemic has an effect on economic activity all across the world. The Corona virus is the pathogen responsible for COVID-19 disease. Specifically, SARS (co-emerging in Guangdong, China, in 2002), MERS-CoV (appeared in the Middle East in 2012), and SARS are the three coronaviruses in a row of the Corona Virus family to arise, be identified, or be detected earlier (CoV-2 emerged in Wuhan, China, in 2019). Since the emergence of COV-2 and its rapid increase in human infection, mathematical modeling on COVID-19 has received attention. Coronavirus research has been ongoing for decades. For more details one can see the recent work published [Tian et. al. [1], Hyun et. al. [2] and Chu et. al. [3] ]

Entry into a nation in contravention of immigration regulations is referred to as illegal immigration. It concerns a number of political, economic, and social issues and has gained significant attention in both industrialized and more developed developing countries due to the controversy it has caused. There are thought to be 280.6 million international immigrants worldwide, according to UN DESA, 2020, which is accessible on the website of the International Organization of Migrants (IOM) [4], while it is challenging to make precise estimates for illegal immigrants. In addition, the majority of illegal immigrants enter a nation lawfully but remain there longer than allowed without having their resident visas reinstated. The non-linear mathematical model used in this proposed study aims to explain the dynamics of the Corona virus's spread among people. The proposed study aims to assess and devise the effective strategies to detect and control the disease from further spread and to prognosticate the future course of a disease. It also aims to study the impact of illegal migrants on the spread of infectious diseases like COVID-19. Inclusion of the factor illegal immigrants along with other parameters make our model more realistic.

- [1] Tian H., et al., (2020): An investigation of transmission control measures during the first 50 days of the COVID-19 epidemic in China., Science, 368(6491), 638-642
- Hyun Mo Yang , Luis Pedro Lombardi Junior, Fabio Fernandes Morata Castro,
   Ariana Campos Yang, (2021): Mathematical modeling of the transmission of SARS- CoV-2—Evaluating the impact of isolation in Sao Paulo State (Brazil) and lockdown in Spain associated with protective measures on the epidemic of CoViD-19,, PLoS One.
- [3] Chu D.K., et al., (2020): Physical distancing, face masks, and eye protection to prevent person-to person transmission of SARS-CoV-2 and COVID-19: a systematic review and meta-analysis., The Lancet
- [4] https://www.iom.int/

## Fuzzy Sumudu Transform to Solve Convolution Type Volterra Fuzzy Integro-Differential Equations

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key-words: Fuzzy Sumudu transform, Volterra fuzzy integro-differential equation.

#### Abstract:

In the recent years, the area of fuzzy integro-differential equations has developed a lot and plays a key role in the field of engineering. The elementary impression and arithmetic of fuzzy sets were first introduced by Zadeh. Later, the area of fuzzy derivative and fuzzy integration was studied, and some general results were developed. Furthermore, FIDEs in a fuzzy setting are a natural way to model the ambiguity of dynamic systems. Consequently, different scientific fields, such as physics, geography, medicine, and biology, pay much importance to the solution of different FIDEs. Solutions to these equations can be utilized in different engineering problems. Seikkala first defined fuzzy derivatives, while the concept of integration of fuzzy functions was first introduced by Dubois and Prade. In the 1990's Watugala [5] has introduced a new integral transform called the Sumudu transform. In [1, 3] is proposed the idea of the fuzzy method of transformation of Sumudu to solve fuzzy partial differential and integro-differential equations. The technique of the fuzzy Sumudu transform method for solving a fuzzy convolution Volterra integral equations and the fuzzy integro-differential equations was developed in [2] and [4]. Sumudu transform along with broad applications has been utilized in the area of system engineering and applied physics.

In this paper, we use of fuzzy Sumudu transform for solving the Volterra fuzzy integro-differential equation of first kind with convolution kernel

$$\int_{0}^{x} k_{1}(x-s) \odot w(s)) ds \oplus \int_{0}^{x} k_{2}(x-s) \odot w^{(n)}(s)) ds = g(x),$$
(5)

with the initial conditions

$$w^{(i)}(0) = b_i, \quad i = 0, 1, 2, ..., n - 1,$$
(6)

where  $k_1, k_2 : [a, b] \times [a, b] \to \mathbb{R}$ , are continuous functions on  $E^1$  and  $g, u : [a, b] \to E^1$  are continuous fuzzy-number valued functions and  $b_i, (i = 0, 1, ..., n - 1)$  are constants. The set  $E^1$  is the set of all fuzzy numbers.

By using fuzzy Sumudu transform method, we directly convert Volterra fuzzy integro-differential equation of first kind with convolution kernel into an algebraic equation. Solving this algebraic equation and applying fuzzy inverse Sumudu transform we obtain the exact solution. This method is illustrated by giving of various examples.

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- Abdul Rahman N. A., Ahmad M. Z., (2016), Fuzzy Sumudu transform for solving fuzzy partial differential equations, J. Nonlinear Sci. Appl., vol. 9, pp. 3226 -3239.
- [2] Abdul Rahman N. A., Ahmad M. Z., (2017), Solving Fuzzy Volterra Integral Equations via Fuzzy Sumudu Transform, Appl. Math. Comput. Intell., vol. 6, pp.19-28.
- [3] Georgieva A., (2020), Double Fuzzy Sumudu Transform to Solve Partial Volterra Fuzzy Integro-Differential Equations, *Mathematics*, vol. 8, pp.692.
- [4] Min Kang S., Iqbal Z., Habib M., Nazeer W., (2019), Sumudu Decomposition Method for Solving Fuzzy Integro-Differential Equations, *Axioms, vol.* 74.
- [5] Watugala G. K., (2002), The Sumudu transform for functions of two variables, Math. Eng. Ind., vol. 8. pp.293-302.

## On the Condition Number of the Vandermonde Matrix with the Mock-Chebyshev Nodes

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key-words: Vandermonde matrix, condition numbers, mock-Chebyshev nodes.

#### Abstract:

Vandermonde matrices arise frequently in computational mathematics in problems that require polynomial approximation. The polynomial interpolation problem through data given at a set of distinct interpolation points can be expressed in the monomial basis. This gives rise to a linear system of equations with a Vandermonde matrix. A difficulty with a Vandermonde matrix is that this matrix with real nodes is generally quite ill-conditioned [1], even for not very high orders. This ill-conditioning may vary reasonably with the distribution of the points. The general recommendation is to use the highly non-uniform Chebyshev nodes, but the problem remains what to do if experimental data is available only at equally spaced points. In this case, polynomial interpolation has the well-known drawback that polynomial interpolation with equidistant nodes is unreliable due to the Runge phenomenon and is also numerically ill-conditioned. By taking advantage of the optimality of the interpolation processes on Chebyshev-Lobatto nodes, the mock-Chebyshev subset interpolation is one of the best strategies to defeat the Runge phenomenon [2, 3, 4].

In this study, we investigate numerically the condition number of the Vandermonde matrix and find out that the condition number can be reduced by using mock-Chebyshev nodes, as in the case of using Chebyshev-Lobatto nodes. Moreover, we extend our investigation to a Vandermonde-like matrix by using a basis of Chebyshev polynomials and show numerically that, in this case, the condition number of this matrix can be reduced significantly.

Acknowledgement: This work has been supported by Yildiz Technical University Scientific Research Projects Coordination Unit under project number FYL-2023-5661.

- [1] *Gautschi, W. and Inglese, G.*, (1988), Lower Bounds for the Condition Number of Vandermonde Matrices. Numer. Math. **52** 241–250.
- [2] *J.P. Boyd, F. Xu*, (2009), Divergence (Runge Phenomenon) for least-squares polynomial approximation on an equispaced grid and Mock-Chebyshev subset interpolation. Appl. Math. Comput. **210** 158–168.
- [3] B.A. Ibrahimoglu, (2020), A fast algorithm for computing the mock-Chebyshev nodes. J. Comput. Appl. Math. 373:112336.
- [4] B.A. Ibrahimoglu, (2021), A new approach for constructing mock-Chebyshev grids. Math. Meth. Appl. Sci. 44 14766–14775.

## The Geometry of Almost Bronze Conjugate Connections

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key-words: Almost bronze structure, (conjugate) linear connection, structural and virtual tensor field.

#### Abstract:

In this study, we give some properties of the conjugate connection on an almost bronze structure. We express the almost bronze conjugate connections in terms of structural and virtual tensors from the almost product structure. In addition, the existence of duality between the almost bronze and almost product conjugate connection is investigated.

- [1] Blaga A. M. and Crasmareanu M., (2013), The geometry of product conjugate connections, An. Stiint. Univ. Al. I. Cuza Iaşi Mat. (N. S.), 59(1), 73-84
- [2] Crasmareanu M. and Hretcanu C. E. (2008), Golden differential geometry, Chaos, Solitons and Fractals, 38(5), 1229-1238
- [3] Özkan M. and Tamirci T., (2022), Some properties of complex golden conjugate connectios, *Facta Universitatis-Series Mathematics and Informatics (NIS)*, 37(5), 1007-1020
- [4] Özkan M. and Doğan S., (2022), Almost bronze structures on differentiable manifolds, *Hindawi Journal of Mathematics*, 2022, *Article ID 6940387*, 11
- [5] Yano K. and Kon M., (1984): Structures on Manifolds, Word Scientific, New York

### IA-Automorphisms of Free Metabelian Leibniz Algebra of Rank n

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key-words: Free metabelian leibniz algebra, IA-automorphisms, wreath product

#### Abstract:

Leibniz algebras were first introduced by Loday [7] as a non-antisymmetric version of Lie algebras. A Leibniz algebra L over a field K is a non-associative algebra with multiplication, called bracket,  $[,]: L \times L \longrightarrow L$  satisfying the Leibniz identity

$$[x, [y, z]] = [[x, y], z] - [[x, z], y]$$

for all  $x, y, z \in L$ . If the condition [x, x] = 0 for all  $x \in L$  is satisfied, this identity is equivalent to the Jacobi identity L is called as Lie Algebra. Let X be a set, L(X) be the free non-associative algebra on X over K and  $I_L$  be the two-sided ideal of L(X)generated by the elements

$$[a, [b, c]] - [[a, b], c] + [[a, c], b]$$

for all  $a, b, c \in L(X)$ . Then the algebra  $F(X) = L(X)/I_L$  is a free Leibniz algebra with the free generating set X. Let K be a field of characteristic zero and F be the free Leibniz algebra with the free generating set  $\{x_1, x_2, ..., x_n\}$  over K. We denote by F' and F'' the derived subalgebras of F and F', respectively. F' is the commutator ideal of F and F/F' is a free K-module. F/F'' will denote the free metabelian Leibniz algebra generated by the set  $\{x_1, x_2, ..., x_n\}$ . IAut(F/F'') denotes the IA automorphism group of F/F''.

In this work, we determine the structure of IA-automorphism group of the free metabelian Leibniz algebra of n-rank over a field K of characteristic zero. Also, we describe an explicit matrix form corresponding to the IA-automorphisms of free metabelian Leibniz algebras of rank n.

- [1] Abdykhalikov A.T., Mikhalev A.A and Umirbaev U.U., (2001), Automorphism of Two-Generated Free Leibniz Algebras, Commun. Algebra , 29(7), 2953-2960
- [2] Abdykhalikov A.T., Mikhalev A.A and Umirbaev U.U., (1992), Automorphisms and Derivations of Abelian Extensions of Some Lie Algebras, Abh. Math. Sem. Univ. Hamburg, 62, 43-57
- [3] Cohn P.M., (1964), Subalgebras of Free Associative Algebras, Proc.London Math. Soc. 14, 618-632
- [4] Czerniakiewicz A.J., (1971), (1972), Automorphisms of Free Associative Algebra of Rank Two, I, II., Trans. Amer. Math. Soc., 160, 393-401; 171, 309-315
- [5] Drensky V., Cattaneo G.M.P., (2002), Varieties of Metabelian Leibniz Algebras, Journal of Algebra and Its Appl., 1(1),31-50
- [6] Drensky V, Papistas A.I., (2005), Automorphisms of Free Left Nilpotent Leibniz Algebras and Fixed Points, Commun. Algebra, 33, 2957-2975
- [7] Loday J.L., Prashvili T., (1993), Universal Enveloping Algebras of Leibniz algebras and (co)Homology., Math. Ann., 296, 139-158
- [8] L. Makar-Limanov, (1970), The automorphisms of the free algebra with two generators, English translation: in Functional Anal. Appl., 262-263
- [9] Smel'kin A.L., (1973), Wreath products of Lie Algebras and Their Application in the Theory of Groups, Trans. Moskov. Math. Soc., 29

## **Triple-Composed** S-Metric Spaces

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key-words: Triple-composed S-metric, S-metric, fixed point.

#### Abstract:

Recently, metric fixed-point theory has been extensively studied. This theory was initiated with Banach fixed-point theorem. Some generalizations of this theorem have been investigated under the different approaches. One of them is to generalize the used contractive condition and another approach is to generalize the used metric space. For the second generalization, a lot of generalized metric spaces were defined in the literature. In this context, our main aim is to introduce the notion of a triple-composed S-metric space as a new generalization of both a metric and an S-metric space. Also, we give some basic and topological notions related to triple-composed S-metric space. We investigate some properties of this notion. Finally, we prove some fixed-point theorems on triple-composed S-metric spaces.

- [1] Ayoob I., Chuan N. Z. and Mlaiki N., (2023): Double-composed metric spaces, Mathematics, vol. 11, pp. 1866-1877
- [2] Özgür N. and Taş N., (2023): On S-metric spaces with some topological aspects, *Electronic Journal of Mathematical Analysis* and Applications, vol. 11, no. 2, pp. 1-8
- [3] Sedghi S., Shobe N. and Aliouche A., (2012): A generalization of fixed point theorems in *S*-metric spaces, *Matematicki Vesnik*, vol. 64, no. 3, pp. 258-266
- [4] Sedghi S. and Dung N. V., (2014): Fixed point theorems on S-metric spaces, Matematicki Vesnik, vol. 66, no. 1, pp. 113-124

## **Bipolar Soft Expert Sets on Nearness Approximation Space**

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key-words: Soft sets, expert sets, near soft sets, bipolar set, bipolar soft expert set.

#### Abstract:

Bipolar soft set is provided with two soft sets, one positive and the other negative. By adding the opinions of experts to this set, it is easier to choose which feature is stronger to find the object we want. The concept of near soft set is a concept that can distinguish soft sets with similar properties according to the same equivalence class. These two sets of features and expert opinions can be thought of as a single feature set.

In this article, the concept of a near bipolar soft expert set and its basic properties are introduced, to which the near set features are added to the bipolar soft expert set. In this new set, with the help of features and expert opinions, a new equivalence relation is created and objects are classified. Accordingly, the basic properties of the set can be examined. With the near bipolar soft expert set, practicality will be provided in decision making so that we can find the most suitable object in practice. This new idea is illustrated with real-life examples. Thanks to the near bipolar soft expert set, we make it easy to choose the one closest to the criteria we want in decision making. Among the many given objects, we can find the ones with similar properties with the properties we want with an equivalence relation that we restrict more by using the experts' opinion.

## References

[1] Enginoğlu, S. and Dönmez, H., (2015) On soft expert sets, Journal of New Theory, vol. 9, pp. 69-81.

- [2] Tasbozan, H., Icen, I., Bagırmaz, N. and Ozcan, A.F., (2017), Soft Sets and Soft Topology on Nearness approximation spaces, *Filomat, vol. 31, pp. 4117-4125.*
- [3] Tasbozan, H., and Bagırmaz, N., (2021), Near Soft Continuous and Near Soft JP-Continuous Functions, *Electronic Journal of Mathematical Analysis and Applications*, vol. 9, pp. 166-171.
- [4] Taşbozan, H., (2020), Near Soft Bağlantılılık, Afyon Kocatepe Üniversitesi Fen Ve Mühendislik Bilimleri Dergisi, vol. 20, pp. 815-818.
- [5] Al-Shami and Tareq M., (2021), Bipolar soft sets: relations between them and ordinary points and their applications, *Complexity, vol. 2021.*
- [6] Dalkılıç, O. and Demirtaş, N., (2022), Combination of the Bipolar Soft Set and Soft Expert Set with an Application in Decision Making, Gazi University Journal of Science, vol. 35, pp. 644-657.

## **Deconstructing Fixed Points Theorems in Modular Metric Spaces**

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key-words: Fixed point theorem, modular metric space.

#### Abstract:

We systematically gather general fixed point theorems within modular and connected metric spaces, thoroughly examining their resemblances and disparities through a diverse array of illustrative examples.

- [1] Aamri M, El Moutawakil D: Some new common fixed point theorems under strict contractive conditions, *J. Math. Anal. Appl.* (2002), 270: 181–188. 10.1016/S0022-247X(02)00059-8.
- [2] Azadifar,B., Sadeghi, G., Saadati, R. and Park, C., *Integral type contractions in modular metric spaces*, Journal of Inequalities and Applications 2013, (2013:483)
- [3] Rouzkard, F., Imdad, M. and Nashine, H.K., *New common fixed point theorems and invariant approximation in convex metric spaces*, Bull. Belg. Math. Soc. Simon Stevin 19 (2012), 311–328.
- [4] M. Jleli and B. Samet, A Generalized Metric Space and Related Fixed Point Theorems, Fixed Point Theory and Appl., (2015:61), 14(2015).
- [5] Ben-El-Mechaiekh, H. Spaces and maps approximation and fixed points, Journal of Computational and Applied Mathematics, 113(2000) 283–308
- [6] Turkoglu, D. and Manav, N. Fixed Point Theorems in New Type of Modular Metric Spaces, Fixed Point Theory and Applications (2018), https://doi.org/10.1186/s13663-018-0650-3.

## Deferred Statistical Convergence and Deferred Statistical Continuity in Locally Solid Riesz Spaces

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key-words: Statistical convergence, locally solid Riesz space, deferred mean.

#### Abstract:

A locally solid Riesz space is a Riesz space endowed with a linear topology  $\tau$  that has a base at zero consisting of solid sets [2]. Albayrak and Pehlivan [3] introduced and studied statistical  $\tau$ -convergent sequences in locally solid Riesz spaces. Thereafter, Mohiuddine and Alghamdi [4] extended their idea through a lacunary sequence in locally solid Riesz spaces.

In this study, using deferred density ([1],[5]) we introduce the concepts of deferred statistical  $\tau$ -convergence, deferred statistical  $\tau$ -boundedness and deferred statistical  $\tau$ -Cauchiness of sequences in locally solid Riesz spaces. Along with some inclusion theorems, we also generalize statistical continuity by defining deferred statistical continuity of functions between locally solid Riesz spaces. Let  $(R, \tau)$  be a locally solid Riesz space and  $\langle a, b \rangle \in \Omega$  be given. A sequence  $x = (x_m)$  in R is said to be *deferred statistically*  $\tau$ -convergent to the point  $\sigma \in R$  if for each  $U \in \mathcal{N}_{\tau}(\theta)$ ,  $\mathcal{D}_a^b(M_U) = 0$  where  $M_U = \{m \in N : (x_m - \sigma) \notin U\}$ , i.e.,

$$\lim_{n \to \infty} \frac{1}{b_n - a_n} \sum_{m = a_n + 1}^{b_n} \chi_{M_U}(m) = 0.$$

We denote this by  $S_{\tau}^{\langle a,b\rangle} - \lim x_m = \sigma$ . We shall denote, by  $S_{\tau}^{\langle a,b\rangle}(R)$ , the set of all *deferred statistically*  $\tau$ -convergent sequences in  $(R, \tau)$ .

- [1] Agnew, R.P., 1932: On deferred Cesàro mean, Annals of Mathematics, 33(3), 413-421.
- [2] Roberts, G.T., 1952: Topologies in vector lattices, Math. Proc. Cambridge Philos. Soc., 48, 533-546.
- [3] Albayrak, H. and Pehlivan, S., 2012: Statistical convergence and statistical continuity on locally solid Riesz spaces, *Topol. Appl.*, *159*, *1887-1893*.
- [4] Mohiuddine, S. A. and Alghamdi, M. A., 2012: Statistical summability through a lacunary sequence in locally solid Riesz spaces, *Journal of Inequalities and Applications*, 225, *doi.org/10.1186/1029-242X-2012-225*.
- [5] Küçükaslan, M. and Yılmazturk, M., 2016: On deferred statistical convergence of sequences, *Kyungpook Mathematical Journal*, 56, 357-366.

## An Extended Calculus on the Hopf Superalgebra $O(GL_{h,h'}(1|1))$

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key-words: Hopf superalgebra, bi-covariant differential calculus, cartan-maurer one-forms, quantum vector fields, cartan calsulus.

#### Abstract:

After the discovery of quantum groups [1], the interest in the geometry of quantum Lie (super)groups and Lie (super)algebras in mathematics and mathematical physics has increased considerably and the work on this subject continues at full speed. In the language of quantum groups, there are two types of deformations: standard (or q-deformation) and non-standard (or h-deformation). The q-deformation of Lie groups and algebras are presented in [2] and non-standard deformation in [3-5].

Let  $\mathcal{A}$  be a free superalgebra and  $a, \beta, \gamma, d$  be elements of  $\mathcal{A}$ , where the degrees of a and d are 0 and  $\beta$  and  $\gamma$  are 1. The supergroup  $GL(1|1, \mathcal{A}) := GL(1|1)$  is the group of matrices of the form  $T = \begin{pmatrix} a & \beta \\ \gamma & d \end{pmatrix}$ . The two-parameter h-deformation of the supergroup GL(1|1), denoted by  $GL_{h,h'}(1|1)$ , is given in [5]. The coordinate algebra of this quantum supergroup, denoted by  $O(GL_{h,h'}(1|1))$ , has a Hopf superalgebra structure. So, an extended differential calculus can be constructed on this Hopf superalgebra  $O(GL_{h,h'}(1|1))$ , which will be done in the present work. This calculus includes functions on the quantum supergroup  $GL_{h,h'}(1|1)$ , their differentials, partial derivatives and inner derivatives. Therefore, they must have possible (h, h')-deformed commutation relations between them. To find them, we will use the bi-covariance of the calculus. Then a linear homomorphism  $\sigma : O(GL_{h,h'}(1|1)) \longrightarrow M_4(C)$  exists, where C denotes the complex numbers. Using this description acting on the generators of  $O(GL_{h,h'}(1|1))$ , we obtain matrix representations that preserve the structure of the superalgebra. By defining super Cartan-Maurer 1-forms and finding the relevant relations, we have another algebra homomorphism  $\mu : O(GL_{h,h'}(1|1)) \longrightarrow M_4(C)$ . Using this description, we will find the commutation relations of the inner derivatives with the generators of  $O(GL_{h,h'}(1|1))$ , differentials and partial derivatives. Thus an extended differential calculus on  $O(GL_{h,h'}(1|1))$  will be established.

- [1] Drinfeld V.G., (1986), Quantum groups, Proc. I.C.M. Berkeley, 798-820
- [2] Faddeev L.D., Reshetijhin N.Y. and Takhtajan L.A. (1990), Quantizations of Lie groups and Lie algebras, *Leningard. Math. J.*, 1, 193-225
- [3] Aghamuhammedi A., Khorrami M. and Shariati A., (1995), Jordanian deformation of as a contraction of itd Drinfeld-Jimbo deformation, *J. Phys. A: Math. and Gen.*, 28, 8:L225
- [4] Dabrowski L., Parashar, P., (1996), h-deformation of GL(1|1) Lett. Math. Phys., 38, 331-336
- [5] Celik S., (1997), Two parametric extension of h-deformation of GL(1|1), Let. Math. Phys., 42 (4), 299-308

## Some Permutation and Complete Permutation Polynomials over Finite Fields

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keywords: Permutation polynomial, finite fields, absolutely irreducible.

#### Abstract:

Let  $\mathbb{F}_q$  be a finite field with q elements, where q is a power of a prime. A polynomial  $g(x) \in \mathbb{F}_q[x]$  is called a *permutation* polynomial (PP) over  $\mathbb{F}_q$  whenever the associated function  $g: a \mapsto g(a)$  is a permutation of  $\mathbb{F}_q$ . Recently there has been a great interest in permutation polynomials with a few terms because of their simple algebraic structures and extraordinary properties. Permutation polynomials are also very important since there are many applications of them in areas such as cryptography, coding theory and combinatorial designs. In this talk we will try to give our recent results on permutation polynomials over finite fields.

- [1] Akbal Y., Gülmez Temür B. and Ongan P., (2020), A short note on permutation trinomials of prescribed type, *Comm. Algebra*, 48(4), 1608-1612
- [2] Ongan P., Gülmez Temür B., (2019), Some permutations and complete permutation polynomials over finite fields, *Turkish Journal of Mathematics*, 43(5)
- [3] Ongan P., Gülmez Temür B., (2020), A specific type of permutation and complete permutation polynomials over finite fields, J. Algebra Appl., 19(4)
- [4] Özbudak F., Gülmez Temür B., (2021), Classification of permutation polynomials of the form  $x^3g(x^{q-1})$  of  $\mathbb{F}_{q^2}$  where  $g(x) = x^3 + bx + c$  and  $b, c \in \mathbb{F}_q^*$ , Des. Codes Cryptogr. 90(7), 1537-1556
- [5] Özbudak, F., Gülmez Temür B., (2023), Classification of some quadrinomials over finite fields of odd characteristic, *Finite Fields and Their Applications*, 87, 102158
- [6] Özbudak F., Gülmez Temür B., (2023), Complete characterization of some permutation polynomials of the form  $x^r(1 + ax^{s_1(q-1)} + bx^{s_2(q-1)})$  over  $\mathbb{F}_{q^2}$ , Cryptography and Communications, DOI:10.1007/s12095-023-00641-7
## Overview of Different Types of Stability of the Solutions of Differential Equations with Non-instantaneous Impulses

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key-words: Differential equations, non-instantaneous impulses, stability, Lyapunov functions.

#### Abstract:

In this talk, we study nonlinear differential equations with non-instantaneous impulses. Several types of stability of the solutions such as stability, uniform stability, and Lipschitz stability, are presented, discussed and illustrated on examples. Appropriate modification of the classical Lyapunov function is defined and applied, since the solutions of the differential equations with non-instantaneous impulses are piecewise continuous functions. Some sufficient conditions are presented and verified on the examples. The computer algebra system "Wolfram Mathematica" is used to solve the given differential equations and to graph their solutions.

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## An Approach to Schur Stable Matrix Families

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Key-words: Schur stability, continuity theorem, matrix family.

#### Abstract:

This study is concerned with the Schur stability of the matrix families  $\mathcal{L}(A, B)$  consists of linear sum and  $\mathcal{C}(A, B)$  consists of convex combination. According to Lyapunov's theorem, the matrix A is Schur stable if and only if there is a symmetric and positive solution H which satisfy the Lyapunov matrix equation  $A^*HA - H + I = 0$  [1].  $\omega(A) = ||H||$  is defined as the quality of Schur stability of the matrix A [2, 3]. For the matrices  $A \in S_N = \{A \in M_N(\mathbb{C}) \mid \omega(A) < \infty\}, B \in M_N(\mathbb{C})$ , the Schur stability intervals  $\mathcal{I}_{\mathcal{L}}$  and  $\mathcal{I}_{\mathcal{C}}$  of the matrix families were determined using the continuity theorems, respectively. The continuity theorems based on Schur stability parameters  $\omega$  indicates the sensitivity of the Schur stability [4, 5]. The intervals  $\mathcal{I}_{\mathcal{L}}$  and  $\mathcal{I}_{\mathcal{C}}$  were extended with the help of methods which based on continuity theorems and the algorithms which based on methods. At the end of, the extended intervals  $\mathcal{I}_{\mathcal{L}}^e$  and  $\mathcal{I}_{\mathcal{C}}^e$  were obtained, which guarantees the Schur stability of the matrix families. On the other hand, the similar results were done for  $\omega^*$ -Schur stability. Finally, the examples related to the Schur stability intervals were given.

- [1] Bulgak H., (1999), Pseudoeigenvalues, spectral portrait of a matrix and their connections with different criteria of stability, *Error Control and Adaptivity in Scientific Computing*, pp. 95-124.
- [2] Bulgakov A. Ya. and Godunov S. K., (1988) Circle dichotomy of the matrix spectrum, Siberia Math. J., 29(5), pp. 59-70.
- [3] Aydın K., Bulgak H. and Demidenko G. V., (2000) Numeric characteristics for asymptotic stability of solutions to linear difference equations with periodic coefficients, *Siberian Math. J., vol. 41(6), pp. 1005-1014.*
- [4] Aydın K., Bulgak H. and Demidenko G. V., (2002), Asymptotic stability of solutions to perturbed linear difference equations with periodic coefficients, *Siberian Mathematical Journal*, 43(3), pp. 389-401.
- [5] Duman A. and Aydın K., (2011) Sensitivity of Schur stability of systems of linear difference equations with constant coefficients, *Scientific Research and Essays*, 6(28), pp. 5846-5854.

## **On a Modification of Mellin Convolution Type Operators**

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key-words: Mellin integral operators, Voronovskaya type theorem, logarithmic Taylor formula.

#### Abstract:

In this talk, we give a modification of Mellin convolution type operators. In this way, we obtain the rate of convergence with the modulus of the continuity of the m-th order Mellin derivative of function f, but without the derivative of the operator. After that, we express Taylor formula including Mellin derivatives with integral remainder. Later, a Voronovskaya type theorem is proved. In the last part, we state weighted approximation properties of the modified operator.

- [1] Aral, A., Acar, T. and Kursun, S., (2022), Generalized Kantorovich forms of exponential sampling series, Anal. Math. Phys., 12 (2), 1-19.
- [2] Angeloni, L., Vinti, G., (2014), Convergence and rate of approximation in BV  $\phi$  (RN) for a class of Mellin integral operators, *Rend. Lincei-Mat. Appl.*, 25 (3), 217-232.
- [3] Angeloni, L., Vinti, G., (2015), Approximation in variation for Mellin integral operators, PAMM, 15 (1), 649-650.
- [4] Bardaro, C., Mantellini, I., (2011), A note on the Voronovskaja theorem for Mellin-Fejer convolution operators, Appl. Math. Lett., 24 (12), 2064-2067.
- [5] Bardaro, C., Mantellini, I., (2022), Boundedness properties of semi-discrete sampling operators in Mellin–Lebesgue spaces, Mathematical Foundations of Computing, 5 (3), 219-229.
- [6] Bardaro, C., Mantellini, I., (2006), Pointwise convergence theorems for nonlinear Mellin convolution operators, Int. J. Pure Appl. Math., 27 (4), 431-447.
- [7] Butzer, P.L., Jansche, S., (1997), A direct approach to the Mellin transform, J. Fourier Anal. Appl., 3, 325-375.
- [8] Butzer, P.L., Nessel, R.J., (1971): Fourier Analysis and Approximation I, Academic Press, New York.
- [9] Bardaro, C., Mantellini, I., (2007), Voronovskaya-type estimates for Mellin convolution operators, Res. Math., 1 (50), 1-16.
- [10] Bardaro, C., Mantellini, I., (2011), Approximation properties for linear combinations of moment type operators, Comput. Math. Appl., 62 (5), 2304-2313.
- [11] Bardaro, C., Mantellini, I., (2011), Asymptotic behaviour of Mellin-Fejer convolution operators, East J. Approx., 17 (2), 181-201.
- [12] Bardaro, C., Mantellini, I., (2012), On the iterates of Mellin-Fejer convolution operators, Acta Appl. Math., 121 (1), 213-229.
- [13] Bardaro, C., Mantellini, I., (2014), On Mellin convolution operators: a direct approach to the asymptotic formulae, Integral Transf. Spec. Funct., 25 (3), 182-195.
- [14] Bardaro, C., Mantellini, I. and Tittarelli, I., (2023), Convergence of semi-discrete exponential sampling operators in Mellin-Lebesgue spaces, *Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat.*, 117 (30).
- [15] Bertero, M., Pike, E., (1991), Exponential-sampling method for Laplace and other dilationally invariant transforms: I. Singular-system analysis, *Inverse Probl.*, 7 (1).
- [16] Bertero, M., Pike, E., (1991), Exponential-sampling method for Laplace and other dilationally invariant transforms: II. Examples in photon correlation spectroscopy and fraunhofer diffraction, *Inverse Probl.* 7 (1).
- [17] Butzer, P.L., Jansche, S., (1998), The exponential sampling theorem of signal analysis, Atti Semin. Mat. Fis. Univ. Modena, 46, 99-122.
- [18] Butzer, P.L., Jansche, S., (1997), A direct approach to the Mellin transform, J. Fourier Anal. Appl., 3 (4), 325-376.
- [19] Kolbe, W., Nessel, R.J., (1972), Saturation theory in connection with Mellin transform methods, SIAM J. Math Anal., 3 (2), 246-262.
- [20] Mamedov, R., (1991): The Mellin Transform and Approximation Theory, Elm, Baku.

## **On the Invariant Submanifolds of Almost Bronze Riemannian Manifolds**

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key-words: Bronze means, almost bronze structure, almost bronze Riemannian manifold, invariant submanifold.

#### Abstract:

Kalia introduced a new bronze ratio in [3] which is related to the bronze Lucas and bronze Fibonacci numbers, which are not members of the metallic mean family. In [5] the authors investigated new almost bronze structures by using the new bronze mean. The new almost bronze structures are polynomial structures with a structure polynomial of  $Q(\Phi) = (\Phi)^2 - m\Phi + Id$  for  $m \in \mathbb{R} \setminus [-2, 2]$  on differentiable manifolds. They used an almost product structure to examine a new almost bronze structure's geometry on a differentiable manifold and defined an almost bronze Riemannian manifold.

In this paper, we study the geometry of submanifolds of almost bronze Riemannian manifolds. We give fundamental properties of structure induced on submanifolds and examine the invariant submanifolds of almost bronze Riemannian manifolds.

- [1] Hretcanu C. E., Blaga A. M., (2018), Submanifolds in metallic Riemannian manifolds, Differential Geometry-Dynamical System, 20, 83-97
- [2] Hretcanu C. E., Crasmareanu M., (2007), On some invariant submanifolds in a Riemannian manifold with golden structure, An. Stiint. Univ. Al. I. Cuza Iasi. Mat.(NS), 53(1), 199-211
- [3] Kalia S. The generalizations of the Golden ratio: Their powers, continued fractions, and convergents., Available online: http://math.mit.edu/research/highschool/primes/papers.php
- [4] Kılıç E., Gök M., Keleş S., (2020), Invariant submanifolds in golden Riemannian manifolds, Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, 69(2), 1119-1132
- [5] Özkan M., Doğan S., (2022), Almost bronze structures on differentiable manifolds, Journal of Mathematics, 2022, Article ID 6940387, 11

## A Variational Problem on the Dual Pseudo-sphere

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key-words: Dual Euler-Lagrange equations, dual geodesic curvature, dual pseudo-spherical elastica, elastic strips with non-null directrix, dual pseudo-sphere.

### Abstract:

We aim to find an action of a dual bending energy functional defined on the dual pseudo-sphere or known Lorentzian dual unit sphere in dual Lorentzian 3-space. We express the necessary condition for the non-null dual curve on the dual pseudo-sphere to be elastic by a dual Euler-Lagrange equation. When solving the real part of the dual Euler-Lagrange equation using Jacobi elliptic functions, we solve the dual part using the method of integral factors. We classify the solutions of dual spacelike and timelike curves on the dual pseudo-sphere. Additionally, we are employing the Study mapping to investigate the scenarios where any timelike or spacelike elastic curve on the dual pseudo-sphere corresponds to an elastic strip in Minkowski 3-space.

- Ayyıldız N., Çöken A.C. and Yücesan A., (2007), A characterization of dual Lorentzian spherical curves in the dual Lorentzian space, *Taiwanese Journal of Mathematics*, 11(4), 999-1018
- [2] Abdel-Baky R., (2017), Evolutes of hyperbolic dual spherical curve in dual Lorentzian 3-space, International Journal of Analysis and Applications, 15(2), 114-124
- [3] Choi S.M., (1995), On the Gauss map of ruled surfaces in a 3-dimensional Minkowski space, Tsukuba journal of mathematics, 19(2), 285-304
- [4] Liu H. and Yuan Y., (2012), Pitch functions of ruled surfaces and B-scrolls in Minkowski 3-space., Journal of geometry and physics, 62(1), 47-52
- [5] Guggenheimer, H.W. (2012): Differential geometry, Courier Corporation
- [6] Oral M., (2010): Elastic curves on hyperquadrics in Minkowski 3-space, M.Sc. Thesis, Süleyman Demirel University
- [7] Scheaf J.A., (1988): Curvature theory of line trajectories in spatial kinematics, Ph.D. Thesis, University of California
- [8] Schwalm W.A., (2015): Lectures on selected topics in mathematical physics: Elliptic functions and elliptic integrals, IOP Publishing, Morgan-Claypool Publishers
- [9] Tükel G.Ö. and Yücesan A., (2019), Elastic strips with timelike directrix, Mathematical Reports, 21(1), 67-83
- [10] Tükel G.Ö. and Yücesan A., (2019), Elastic strips with spacelike directrix, Bulletin of the Malaysian Mathematical Sciences Society, 42, 2623-2638
- [11] Uğurlu H.H. and Çalişkan A., (1996), The Study Mapping for Directed Space-Like and Time-Like in Minkowski 3-Space R<sub>1</sub><sup>3</sup>, Mathematical and Computational Applications, 1(2), 142-148
- [12] Veldkamp G.R., (1976), On the use of dual numbers, vectors and matrices in instantaneous, spatial kinematics, Mechanism and Machine Theory, 11(2), 141-156
- [13] Yaylı Y., Çalişkan A. and Uğurlu H.H., (2002), The E. Study maps of circles on dual hyperbolic and Lorentzian unit spheres  $H_0^2$  and  $S_1^2$ , *Mathematical Proceedings of the Royal Irish Academy*, 102A(1), 37-47
- [14] Yücesan A. and Tükel G.Ö., (2021): Elastic curves in the dual Lorentzian space, International Modern Scientific Research Congress, İstanbul, 884-892
- [15] Yücesan A. and Tükel G.Ö., (2023), Dual spherical elastica, *Filomat*, 37(8), 2483-2493

## **Curves Along Lorentzian Submersions**

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key-words: Lorentzian submersion, second fundamental form, geodesic, circle, helix.

#### Abstract:

We explore the impact of curve theory on the characterization of Lorentzian submersions which are special cases of semi-Riemannian submersions. We firstly give a classification of curves according to their causal character along a Lorentzian submersion. Then, we investigate the effects of certain curves, specifically the non-null Frenet curve, the non-null circle, the non-null helix, and the geodesic, under a Lorentzian submersion, and we seek their geometric meaning. Finally, we examine the situation where a Cartan-framed lightlike curve is carried from the total manifold to the base manifold through a Lorentzian submersion.

- Akyol M.A. and Gündüzalp Y., (2018), Semi-invariant semi-Riemannian submersions, Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, 67(1), 80-92
- [2] Allison D., (1996), Lorentzian Clairaut submersions, Geometriae Dedicata, 63, 309-319
- [3] Baditoiu G., (2004), Semi-Riemannian submersions with totally geodesic fibres, Tohoku Mathematical Journal, Second Series, 56(2), 179-204
- [4] Bishop R.L., (1972): Clairaut submersions, in Differential Geometry in Honor of K. Yano, Kinokuniya, Tokyo, pp. 21-31
- [5] Falcitelli M., Ianus S. and Pastore A.M., (2004): Riemannian submersions and related topics, World Scientific, River Edge NJ
- [6] Gray A., (1967), Pseudo Riemannian almost product manifolds and submersions, Journal of Mathematics and Mechanics, 16(7), 715-737
- [7] Huang R.P. and Shang D.H., (2009), Generalized elastic curves in the Lorentz flat space L<sup>4</sup>, Applied Mathematics and Mechanics, 30(9), 1193-1200
- [8] Ikawa T., (1981), On some curves in Riemannian geometry, Soochow Journal of Mathematics, 7, 44
- [9] Ikawa T., (1985), On curves and submanifolds in an indefinite-Riemannian manifold, Tsukuba journal of mathematics, 9(2), 353-371
- [10] Kim Y.H. and Yoon D.W., (2000), Ruled surfaces with finite type Gauss map in Minkowski spaces, Soochow Journal of Mathematics, 26(1), 85-96
- [11] Magid M.A., (1981), Submersions from anti-de Sitter space with totally geodesic fibers, Journal of Differential Geometry, 16(2), 323-331
- [12] Nomizu K. and Yano K., (1974), On circles and spheres in Riemannian geometry, Mathematische Annalen, 210(2), 163-170
- [13] O'neill B., (1983): Semi-Riemannian geometry with applications to relativity, Academic press
- [14] O'neill B. (1966), The fundemantal equations of a submersion, Michigon Mathematical Journal, 13, 459-469
- [15] Şahin B., (2017): Riemannian submersions, Riemannian maps in Hermitian geometry, and their applications, Elsevier
- [16] Siddiqi M., Khan M.A., Ishan A.A. and Chaubey S.K. (2022), Anti-invariant Lorentzian submersions from Lorentzian concircular structure manifolds, *Frontiers* in *Physics*, 10, 31
- [17] Tukel G. O., Sahin B. and Turhan T. (2023), Certain curves along Riemannian submersions, Filomat, 37(3), 905-913

## **On Unbounded Order Continuous Urysohn Operators**

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key-words: Riesz space, unbounded order convergence, unbounded order continuous Urysohn operator.

#### Abstract:

Let E and F be two Archimedean Riesz spaces (vector lattices). A function  $T : E \to F$  is called orthogonally additive if T(x + y) = Tx + Ty for every disjoint elements  $x, y \in E$ . An orthogonally additive order bounded operator  $T : E \to F$  is called an Urysohn operator and the space of all Urysohn operators from E to F is denoted by U(E, F). A net  $(x_{\alpha})$  in E is said to be order convergent to  $x \in E$ , if there is a net  $(y_{\beta})$  in E with  $y_{\beta} \downarrow 0$  and that for every  $\beta$ , there exists  $\alpha_0$  satisfying  $|x_{\alpha} - x| \leq y_{\beta}$  for all  $\alpha \geq \alpha_0$ , and it is denoted by  $x_{\alpha} \xrightarrow{\circ} x$ . A net  $(x_{\alpha})$  in E is unbounded order convergent to  $x \in E$  if  $|x_{\alpha} - x| \land u \xrightarrow{\circ} x$  for all  $u \in E_+$ , and this convergence is denoted by  $x_{\alpha} \xrightarrow{u_{\alpha}} x$ . In this study, we introduce a new class of operators in Riesz spaces. We say that an Urysohn operator T from Riesz space E to Riesz space F is an unbounded order continuous (*uo*-continuous) Urysohn operator if  $x_{\alpha} \xrightarrow{u_{\alpha}} x$  in E implies  $Tx_{\alpha} \xrightarrow{u_{\alpha}} Tx$  in F, and we denote by  $U_{u_{\alpha}}(E, F)$ . We get a lattice calculus of  $U_{u_{\alpha}}(E, F)$  in the Riesz space E and a Dedekind complete Riesz space F.

- [1] Aliprantis C.D. and Burkinshaw O.,(2006): Positive Operators, Berlin, Springer.
- [2] Gao N. and Xanthos F. (2014), Unbounded order convergence and application to martingales without probability, *Journal of Mathematical Analysis and Applications*, 415, 931-947.
- [3] Gao N., Troitsky VG. and Xanthos F.,(2017), Uo-convergence and its applications to Cesaro means in Banach lattices, *Israel Journal of Mathematics*, 220, 649-689.
- [4] Maz'on J.M. and Segura de Le'on S.,(1990), Order bounded orthogonally additive operators, *Rev.Roumane Math. Pures Appl.* 35, no. 4, 329–353.
- [5] Maz'on J.M. and Segura de Le'on S., (1990), Uryson operators, Rev. Roumane Math. Pures Appl. 35, no. 5, 431-449.
- [6] Mykhaylyuk V. and Popov M.,(2022), ε-Shading Operator on Riesz Spaces and Order Continuity of Orthogonally Additive Operators, *Results Math.*, 77, Paper No. 209.
- [7] Pliev M.A. and Ramdane K.,(2018), Order unbounded orthogonally additive operators in vector lattices. *Mediter. J. Math.* 15(2), Paper No. 55.

## On a Unique Solution of the Generalized Functional Equation Arising in Mathematical Psychology and Theory of Learning Approached by the Fixed Point Method

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key-words: Psychological learning theory, functional equations, fixed points.

### Abstract:

In this work, we intend to investigate several psychological and learning theory models and establish a general functional equation for them. By using the fixed point theory tools, we obtain the results related to the existence, uniqueness, and stability of a solution to the proposed functional equation. Finally, we give two examples to support our main results.

- Turab, A. and Sintunavarat, W., (2019), On analytic model for two-choice behavior of the paradise fish based on the fixed point method, J. Fixed Point Theory Appl., 21, 56.
- [2] Berinde, V. and Khan, A.R., (2015), On a functional equation arising in mathematical biology and theory of learning, *Creat. Math. Informat.*, 24(1), 9–16.
- [3] Hyers, D.H., (1941), On the stability of the linear functional equation, Proc. Natl. Acad. Sci. USA, 27, 222-224.

# On Existence and Multiplicity of Solutions for A Biharmonic Problem with Weights via Ricceri's Theorem

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key-words: p(.)-biharmonic operator, Ricceri's variational principle, variational methods.

### Abstract:

In this work, we consider a special nondegenerate equation with two weights. We investigate multiplicity result of this biharmonic equation. Mainly, our purpose is to get this result using an alternative Ricceri's theorem. Moreover, we give some compact embeddings in variable exponent Sobolev spaces with second order to proof the main idea.

- [1] Aydin I., Unal C., (2020): The Kolmogorov-Riesz theorem and some compactness criterions of bounded subsets in weighted variable exponent amalgam and Sobolev spaces, *Collect. Math.* 71, 349-367.
- [2] Aydin I., Unal C., (2021): Three solutions to a Steklov problem involving the weighted p(.)-Laplacian, Rocky Mountain J. Math. 51(1), 67-76.
- [3] Cui M., Zhang S., (2020): On the uniform convergence of the weak Galerkin finite element method for a singularly-perturbed biharmonic equation, *Journal of Scientific Computing* 82(5).
- [4] Diening L., Harjulehto P., Hästö P., Růžička M., (2011): Lebesgue and Sobolev Spaces with Variable Exponents, Springer-Verlag, Berlin.
- [5] Fan C. M., Huang Y. K., Chen C. S., Kuo S. R., (2019): Localized method of fundamental solutions for solving two-dimensional Laplace and biharmonic equations, *Engineering Analysis with Boundary Elements 101, 188-197.*
- [6] Kefi K., (2019): For a class of p(x)-biharmonic operators with weights, RACSAM 113, 1557-1570.
- [7] Li L., Ding L., Pan W., (2013): Existence of multiple solutions for a p(x)-biharmonic equation, *Electron. J. Differ. Equ.* 2013(139), 1-10.
- [8] Ricceri B., (2000): On a three critical points theorem, Arch. Math. (Basel) 75, 220-226.
- [9] Ricceri B., (2009): A three critical points theorem revisited, Nonlinear Anal. 70, 3084-3089.
- [10] Unal C., Aydin I., (2021): Compact embeddings of weighted variable exponent Sobolev spaces and existence of solutions for weighted p(.)-Laplacian, *Complex Var. Elliptic Equ. 66(10)*, 1755-1773.

## Fractional Trigonometric Korovkin-type Results by Statistical Convergence Based On A Power Series Method

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key-words: Caputo derivative, statistical convergence, trigonometric Korovkin-type approximation.

### Abstract:

In approximation theory, Korovkin-type theorems are well used since they provide us to determine the uniform convergence of positive linear operators to identity by using only three functions  $\{1, x, x^2\}$ . They have been investigated in different function spaces, generally by using different concepts of convergences, by using *q*-calculus and rarely by fractional calculus. In this talk, fractional trigonometric Korovkin-type approximation results will be presented via *P*-statistical convergence which depends on a power series. Also, as an aplication of our theorems, various type examples will be constructed.

- [1] Anastassiou G.A and Duman O., (2010), Fractional trigonometric Korovkin theory in statistical sense, *Serdica Mathematical Journal, vol. 35, pp. 121-136*
- [2] Podlubny I., (1999): Fractional Differential Equations, Academic Press, California, USA
- [3] Ünver M. and Orhan C., (2019), Statistical convergence with respect to power series methods and applications to approximation theory, *Numer. Func. Anal. Opt., vol. 40, pp. 535-547*

## **On Modified Mellin–Picard Operators**

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key-words: Mellin-Picard operator, weighted approximation, Voronovskaya-type asymptotic formulae.

#### Abstract:

We propose two modifications for Mellin-Picard operators. First, we compute moments of modified operators. Then, we discuss weighted approximation and obtain Voronovskaya-type formulae for them in polynomial weighted spaces. An appropriate weighted modulus of continuity is used in order to measure the convergence rate in polynomial weighted spaces. Some graphical examples are presented which are comparing newly obtained operators and old version in case of approximation to the original function. In addition, the modified versions are also compared among themselves in terms of rate of convergence.

- [1] Aral, A., Erbay, H. and Yılmaz, B., 2022, On modified Mellin-Gauss-Weierstrass convolution operators, *Results Math.*, 77, paper no. 130, pp. 1–18.
- [2] Bardaro, C. and Mantellini, I., 2012, The moments of the bivariate Mellin-Picard-type kernels and applications, *Integral Transforms Spec. Funct.*, 23(2), pp. 135–148.
- [3] Bardaro, C. and Mantellini, I., 2014, On Mellin convolution operators: a direct approach to the asymptotic formulae, *Integral Transforms Spec. Funct.*, 25(3), pp. 182–195.
- [4] Butzer, P. L. and Jansche, S., 1997, A direct approach to the Mellin transform, J. Fourier Anal. Appl., 3(4), pp. 325–376.
- [5] Gadziev, A. D., 1974, A problem on the convergence of a sequence of positive linear operators on unbounded sets, and theorems that are analogous to P. P. Korovkin's theorem, (Russian) *Dokl. Akad. Nauk SSSR*, *218*, *pp. 1001–1004*.
- [6] King, J.P., 2003, Positive linear operators which preserve  $x^2$ , Acta Math. Hungar, 99, pp. 203–208.
- [7] Mamedov, R. G., 1991, The Mellin Transform and Approximation Theory, Elm, Baku.
- [8] Yuksel, I. and Ispir, N., 2006, Weighted approximation by a certain family of summation integral-type operators, *Comput. Math. Appl.*, 52(10-11), pp. 1463–1470.

## Ruled Surfaces Created by Successor Frames on the $T_1N_1$ Smarandache Curve Obtained from the Tangent and Normal Vectors of the Successor Curve

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#### Abstract:

In this study, the smarandache ruled surfaces obtained by the movement of the frenet vectors of the successor curve on the  $T_1N_1$  smarandache curve were defined. Then, the Gaussian and mean curvatures of each ruled surface were calculated. It was shown that ruled surfaces formed by the movement of the tangent vector of the successor curve and the vector  $T_1N_1$  along the Smarandache curve  $T_1N_1$  are developable surfaces. Additionally, if the principal curve is a planar curve, the ruled surface formed by the principal normal vector of the successor curve along the Smarandache curve is a developable minimum surface. Conditions for other surfaces to be openable or minimal surfaces were given. Finally, these surfaces were given examples and their shapes were drawn with the Maple 17 program.

- [1] Menninger A., (2014): Characterization of the slant helix as successor curves of the general helix, International Electronic Journal of Geometry, 2, 84-91.
- [2] Pressley A., (2010): Elementary Differential Geometry, Springer Science and Business Media.
- [3] 3. Ali A. T., (2010): Special Smarandache curves in the euclidean space, International Journal of Mathematical Combinatorics, 2, 30-36.
- [4] Struik D. J., (1961): Lectures on classical differential geometry, Addison-Wesley Publishing Company.
- [5] Pottmann H., Eigensatz M., Vaxman A., and Wallner J., (2015): Architectural Geometry, Computers and Graphics, 47, 145–164.
- [6] Monterde J., (2009): Salkowski curves revisited: A family of curves with constant curvature and non-constant torsion, *Computer Aided Geometric Design*, *3*, 271-278.
- [7] Stillwell J., (2010): Mathematics and Its History, Undergraduate Texts in Mathematics, Third Edition, Springer.
- [8] Akutagawa K. and Nishikawa S., (1990): The Gauss map and space-like surfaces with prescribed mean curvature in Minkowski 3-space, *Tohoku Mathematical Journal, Second Series*, 42, 67-82.
- [9] Taskopru K. and Tosun M.,(2014): Smarandache Curves on S<sup>2</sup>, Bol. Soc. Paran. Mat. 2014, 32(1), 51-59.
- [10] Grilli L., Senyurt S. and Gur Mazlum S., (2020): Gaussian curvatures of parallel ruled surfaces, Applied Mathematical Sciences, 14, 171-183.
- [11] Cetin M., and Kocayiğit H., (2013): On the Quaternionic Smarandache Curves in Euclidean 3-Space, Int. J. Contemp. Math. Sciences, 3, 139–150.
- [12] Masal M., (2018): Curves according to the successor frame in euclidean 3-Space, Sakarya University Journal of Science, 6, 1868-1873.
- [13] Turgut N. and Yılmaz S., (2008): Smarandache curves in Minkowski space-time, International Journal of Mathematical Combinatorics, 3, 51-55.
- [14] Gur Mazlum S., Senyurt S. and Grilli L., (2023): The Invariants of Dual Parallel Equidistant Ruled Surfaces, Symmetry, 15, 206.
- [15] Ouarab S., (2021): Smarandache Ruled Surfaces according to Frenet-Serret Frame of a Regular Curve in E<sup>3</sup>, Hindawi Abstract and Applied Analysis, Article ID 5526536, 8 pages.
- [16] Ouarab S., (2021): Smarandache ruled Surfaces according to Darboux Frame in E<sup>3</sup>, Hindawi Journal of Mathematics, Article ID 9912624, 10 pages.
- [17] Ouarab S., (2021): NC-Smarandache ruled surface and NW-Smarandache ruled surface according to alternative moving frame in E<sup>3</sup>, Hindawi Journal of Mathematics, Article ID 9951434, 6 pages.
- [18] Senyurt S., Canlı D., Can E. and Gur Mazlum S., (2022): Some special Smarandache ruled surfaces by Frenet frame in  $E^3 II$ , Honam Mathematical Journal, 44, 594-617.
- [19] Senyurt S. and Kaya G., (2019): Successor eğrisinin Frenet vektörlerinden elde edilen Smarandache eğrileri, Black Sea 1st International Multidisciplinary Work Congress, pp.318-324.
- [20] Senyurt S., and Sivas S., (2013): An Application of Smarandache Curve, Ordu Univ. J. Sci. Tech., 3, 46-60.
- [21] Massey W. S., (1962): Surfaces of Gaussian curvature zero in Euclidean 3-space, Tohoku Mathematical Journal, Second Series, 1962, 14, 73-79.

# On the Optical Soliton Solutions to Nonlinear Phenomena Arising in Communication Engineering

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key-words: Optical soliton solutions, extended trial function method, Kerr law nonlinearity, optical fibers.

#### Abstract:

In this paper, the perturbed Radhakrishnan-Kundu-Lakshmanan equation, which is a significant model in the engineering field for analyzing data transmission through nonlinear optical fibers, is studied. The equation is considered to construct the effective scheme for the solution of governing model of the nonlinear phenomena in optical fiber communications. The extended trial function method is implemented to construct optical soliton solutions of the equation under the effect of Kerr law nonlinearity. The results are illustrated by figures with the aid of symbolic computation tools.

- [1] Kudryashov N. A., (2022), Governed optical solitons of the generalized Schrödinger equation with dual-power law of refractive index, *Optik International Journal for Light and Electron Optics, vol. 266, pp.169619.*
- [2] Yu W., Liu W., Triki H., Zhou Q., Biswas A., (2019): Phase shift, oscillation and collision of the anti-dark solitons for the (3+1)-dimensional coupled nonlinear Schrödinger equation in an optical fiber communication system, *Nonlinear Dynamics*, vol. 97, pp. 1253-1262.
- [3] Seadawy A.R., Bilal M., Younis M., Rizvi S.T.R., Makhlouf M.M., Althobaiti S., (2021): Optical solitons to birefringent fbers for coupled Radhakrishnan–Kundu–Lakshmanan model without four-wave mixing, *Optical and Quantum Electronics, vol. 53*, pp. 324-345.

## **Proximity on Digital Topological Spaces**

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key-words: Digital topology, proximity, adjacency relation

#### **Abstract:**

A digital topological space X is a subset of  $\mathbb{Q} \times \mathbb{Q}$  together with a map  $Imq: X \to \mathbb{R}$  called a *digital image on X*. For simplicity, we consider only grayscale images. Each element p in X (aka (sub-)pixels) in a grayscale image has a lumens value in  $\mathbb{R} \in [0, 1]$ . One can also consider time-ordered sequences of images  $Img_t$  in X called *video frames* so that each picture element  $p = (p_1, p_2, t)$ in a video frame at an elapsed time t is called (sub-)voxels.



Figure 1: Sample subimage centered on a voxel p

A picture element with integer coordinates is called a *pixel* (or a *voxel* in a video frame) and Imq(p), is the average value of the square  $\{p_1 \pm \frac{1}{2}\} \times \{p_2 \pm \frac{1}{2}\}$  (see, e.g., sketch a subimage Fig. 1). We extend the results from L. Boxer [1] for a digital image in  $\mathbb{Z} \times \mathbb{Z}$  by introducing new types of adjacency relations. Further, given a digital image on X, we define a proximal relation on the collection of its subimages as well as a proximal relation on the sequence of video frames.

## References

[1] Boxer L., (1999), A classical construction for the digital fundamental group. Journal of Mathematical Imaging and Vision, 10. pp.51-62.

## **On SIR Models With Fractional Derivatives**

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key-words: Covid-19, mask protection efficiency, fractional SIR model.

#### Abstract:

As is known, Covid-19 is one of the most serious epidemic that effects whole world in the recent times. The process of pandemic which is occured because of this epidemic showed us the importance of the mask, distance and hygiene while we contend with this epidemic. Also wearing the efficient mask and wearing the mask conveniently are as important as wearing mask that is one of the most important factor for being protected from Covid-19. On the other hand, wearing mask has the same importance for both susceptible and infected people. In this work, different rates of wearing mask and rates of mask efficiency are taken by using real data in Turkey from April 21,2021 to April 30,2021 for emphasize to importance of these. A fractional SIR model with Caputo fractional derivative is proposed to describe the dynamics of Covid-19 according to wearing mask by using real data in Turkey. The obtained results can give an idea to people who study at this field for add on different parameters to the epidemic models for different epidemics.

- [1] Tomášek P., (2023), On Euler methods for Caputo fractional differential equations, Archivum Mathematicum, vol.59, pp. 287-294.
- [2] Kumar R. and Kumar S., (2014), A New Fractional Modelling on Susceptible Infected-Recovered Equations with Constant Vaccination Rate, Nonlinear Engineering, vol.3, pp. 11-19.
- [3] Odibat Z.M., Shawagfeh N.T., (2007), Generalized Taylor's formula, Computational & Applied Mathematics, vol. 186, pp. 286-293.
- [4] Lin W., (2007), Global existence theory and chaos control of fractional differential equations, Journal of Mathematical Analysis and Applications, vol.332, pp. 709-726.
- [5] Ju J.T.J., Boisvert L.N., Zuo Y.Y., (2021), Face masks against COVID-19: Standards, efficacy, testing and decontamination methods, Advances in Colloid and Interface Science, vol.292, 102435.
- [6] Atanackovic T.M., Pilipovic S., Stankovic B., Zorica D., (2014): Fractional Calculus with Applications in Mechanics : Wave Propagation, Impact and Variational Principles, John Wiley & Sons
- [7] Baleanu D., Diethelm K., Scalas E., Trujillo J.J., (2016): Fractional Calculus : Models and Numerical Methods, World Scientific
- [8] Baleanu D., Güvenç Z.B., Machado J.A.T., (2010): New Trends in Nanotechnology and Fractional Calculus Applications, Springer
- [9] Das S., (2008): Functional Fractional Calculus for System Identification and Controls, Springer
- [10] Lorenzo C.F., Hartley T.T., (2017): The Fractional Trigonometry : With Applications to Fractional Differential Equations and Science, John Wiley & Sons
- [11] Podlubny I., (1999): Fractional Differential Equations : An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of their Solution and some of their Applications, Academic Press
- [12] Schiff J.L. , (1999): The Laplace Transform : Theory and Applications , Springer
- [13] Allen L.J.S , (2007): An Introduction To Mathematical Biology , Pearson Education
- [14] Brauer F., Driessche P.V.D., Wu J., (1945): Mathematical Epidemiology, Springer

## Artificial Neural Network using Quiver Representation of Finite Cyclic Group

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key-words: Quiver representation, finite cyclic group, group algebra, moduli space, dimension.

#### Abstract:

In this paper, we propose the use of quiver representations as a tool for understanding artificial neural network algorithms. Specifically, we construct these algorithms by utilizing the algebra group of a finite cyclic group as vertices, and convolution transformations as maps. We demonstrate that circulant matrices can be used to represent these transformations, and we use this representation to obtain change of basis groups. Furthermore, we examine the properties of moduli spaces, which are formed by the actions of the change of basis group on the set of quiver representations. Through this analysis, we are able to compute the dimension of the moduli spaces.

- [1] Armenta M.A., Jodoin P.M., (2020), The Representation Theory of Neural Networks, arXiv, 2007, 2007.12213
- [2] Wanditra L.C., Muchtadi Alamsyah I., Rachmaputri G., (2020), Wave Packet Transform on Finite Abelian Group, *Southeast Asian Bulletin of Mathematics*, 44, 843–857
- [3] Armenta M.A., Brüstle T., Hassoun S., and Reineke M., (2022), Double framed moduli spaces of quiver representations, *Linear Algebra and its Applications*, 650, 98–131

# Algebraic and Geometric Properties of a Family of Rational Curves

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key-words: Quaternion, curves, basis.

#### Abstract:

This paper consists of two components - an application part and a theoretical part, where the former targets the applications of geometric techniques in generating parametric curves, and the latter focuses on the algebraic analysis of rational space curves. At the application level, we construct a family of rational space curves via quaternion products of two generating curves. At the theoretical level, we use algebraic method to extra a  $\mu$ -basis for this family of curves, and describe a basis for a special submodule of the syzygy module in terms of a  $\mu$ -basis for the syzygy module of this family of curves. A commutative diagram is provided to summarize these results.

- [1] Adkins W., Weintraub S., (1992): Algebra Graduate Texts in Mathematics 136, Springer-Verlag
- [2] Cox D., (2001), Equations of parametric curves and surfaces via syzygies, Contemporary Mathematics, 286, 1-20
- [3] Cox D., Little J. and O'Shea D., (1998): Using Algebraic Geometry, Graduate Texts in Mathematics 185, Springer, New York
- [4] Eisenbud D., (2005): The Geometry of Syzygies, Graduate Texts in Mathematics, 229, Springer
- [5] Evans E.G., Griffith P., (1985), Syzygies, London Mathematics Society Lecture Notes Series, 106
- [6] Goldman R., (2010): Rethinking Quaternions: Theory and Computation, Synthesis Lectures on Computer Graphics and Animation, ed. Brian A. Barsky, No. 13. San Rafael: Morgan & Claypool Publishers
- [7] Wang H., Goldman R., (2019), Surfaces of revolution with moving axes and angles, *Graphical Models*, 106, https://doi.org/10.1016/j.gmod.2019.101047

## The Solution of Interval Linear Equation System by Iterative Decreasing Dimension Method

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key-words: Interval linear systems, iterative decreasing dimension method.

#### Abstract:

Solving the systems of linear equation is a basic problem in linear algebra. In applications, the use of approximate solution methods and calculation errors cause to encounter intervals. Therefore, in this study, we have considered the interval linear equation systems

Ax = b,

(7)

where  $A = (a_{ij})$  is an interval  $n \times n$  -matrix,  $b = (b_i)$  is an interval *n*-vector. Calculating the solution of system (7) is a challenging problem and it has received attention for years (see for example [1], [2], [3]).

In this study, we discussed the solution of the interval linear equation system (7) using the Iterative decreasing dimension method (iddm). Iterative decreasing dimension method was given by [4] and it is an iterative method that calculates the solution of systems of linear algebraic equations. Kaucher interval arithmetic was used in calculations, which extends and completes the classical interval arithmetic algebraically [5].

- [1] Moore R.E., Kearfott R.B. and Cloud M.J., (2009): Introduction to Interval Analysis, SIAM, Philadelphia, PA
- [2] Shary S.P., (2019): Numerical computation of formal solutions to interval linear systems of equations, *CoRR*, *abs/1903.10272*, http://arxiv.org/abs/1903.10272
- [3] Nirmala, T.,Ganesan, K.(2019): Solution of interval linear system of equations-an iterative approach, *AIP Conference Proceedings*, *vol.2112*(1), https://pubs.aip.org/aip/acp/article-pdf/doi/10.1063/1.5112290/14184787/020105\_1\_online.pdf
- [4] Keskin, T., Aydin K., (2007): Iterative decreasing dimension algorithm, Comput. Math. Appl., vol. 53(7), pp. 1153-1158
- [5] Kaucher E., (1980), Interval Analysis in the Extended Interval Space IR, Computing Supplement., vol. 2, pp. 33-49

## **On Gaussian Leonardo Hybrid Polynomials**

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key-words: Hybrid numbers, hybrid polynomials, Leonardo numbers, Leonardo polynomials, Gaussian Leonardo polynomials, Gaussian Leonardo hybrid polynomials.

#### Abstract:

In this study, using the Leonardo polynomials, we define the Gaussian Leonardo polynomials. Next, using the Gaussian Leonardo polynomials and hybrid numbers, we define the Gaussian Leonardo hybrid polynomials. Furthermore, we give some particular properties of the Gaussian Leonardo polynomials and hybrid polynomials.

## References

[1] Alp Y. and Kocer E. G., (2021), Hybrid Leonardo numbers, Chaos Solitons Fractals, 150, 111128

- [2] Catarino P. and Borges A., (2019), On Leonardo numbers, Acta Math. Univ. Comen., 89 (1), 75-86
- [3] Kara N. and Yilmaz F., (2023), On hybrid numbers with Gaussian Leonardo coefficients Mathematics, 11, 1551
- [4] Kürüz F., Dagdeviren A. and Catarino P., (2021), On Leonardo Pisano hybrinomials Mathematics, 9, 2923
- [5] Ozdemir M., (2018), Introduction to hybrid numbers Adv. Appl. Clifford Algebras, 28, 1-32

## A New Decomposition of Continuity via the Localization

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key-words: Ideal topological space, localization, continuity.

#### Abstract:

In general topology, the localization is an important concept such as continuity. So, we focus on a generazition of Kuratowski's local function. To obtain a new decomposition of continuity using it, we introduce some new set types and continuous functions of them. Finally, we give a decomposition of continuity.

- [1] Al-Omari A., Noiri T., (2013), Local closure functions in ideal topological spaces, *Novi Sad Journal of Mathematics 43 (2)* 139–149.
- [2] Dontchev J., (1999), Idealization of Ganster–Reilly decomposition theorems, https://arxiv.org/abs/math/9901017v1.
- [3] Hatir E., Noiri T., (2002), On decompositions of continuity via idealization, Acta Math. Hungar., 96, 341--349.
- [4] Jankovic D., Hamlett T. R., (1992), Compatible extensions of ideals, Boll. Un. Mat. Ital. 6, 453-465.
- [5] Jankovic D., Hamlett T.R., (1990), New topologies from old via ideals, The American Mathematical Monthly, 97(4), 295-310.
- [6] Kuratowski K., (1966), Topology Volume I, Academic Press, New York-London.
- [7] Mashhour A. S., Abd El-Monsef M. E., El-Deeb S. N., (1982), On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53, 47–53.
- [8] Njamcul A., Pavlovic A., (2021), On closure compatibility of ideal topological spaces and idempotency of the local closure function, *Period. Math. Hung.*, https://doi.org/10.1007/s10998-021-00401-1.
- [9] Pavlovic A., (2016), Local function versus local closure function in ideal topological spaces, Filomat (30) 14, 3725–3731.
- [10] Tunc, A.N., Ozen Yıldırım S., (2021), New sets obtained by local closure functions, Annals of Pure and Applied Mathematical Sciences, 1 (1), 50–59.
- [11] Velicko N.V., (1968), H-closed topological spaces, American Mathematical Society, 78 (2), 103–118.

## Some Novel Estimations of Hadamard Type Inequalities for Different Kinds of Convex Functions via Tempered Fractional Integral Operator

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key-words: Hadamard type inequalities, tempered fractional integral operators, (h, m)-convex functions, s-convex functions.

### Abstract:

The theory of convexity plays a vital role in different fields of pure and applied mathematics. Consequently the classical concepts of convex sets and convex functions have been generalized in different directions. Among others, (h, m)-convex functions and s-convex functions have been used to provide novel estimations in the literature by several researchers (see the papers [3, 5]). In this paper, some novel estimations of Hadamard type integral inequalities have been obtained for (h, m)-convex functions and s-convex functions by using tempered fractional integral operator (see [1, 2]). The results have been provided by using the assumptions that are given in theorems, the definitions of functions and properties of the fractional integral operator. Several special cases have been considered.

- [1] Li, C; Deng, W; Zhao, L. (2019), Well-posedness and numerical algorithm for the tempered fractional ordinary differential equations, *Discret. Contin. Dyn. Syst.-B*, 24, 1989–2015.
- [2] Meerschaert, M.M; Sabzikar, F; Chen, J. (2015), Tempered fractional calculus, J. Comput. Phys., 293, 14-28.
- [3] Özdemir, M.E, Akdemir, A.O, Set, E. (2016), On (h, m)-convexity and Hermite Hadamard type inequalites, *Transylvanian Journal of Mathematics and Mechanics*, 8, 51-58.
- [4] Mitrinović, D.S., Pečarić, J.E. and Fink, A.M. (1993), Classical and new inequalities in analysis. Mathematics and its Applications, *Kluwer Academic Publishers Group, Dordrecht.*

## New Estimations for Chebyshev Type Inequalities via Generalized Proportional Fractional Integral Operators

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key-words: Chebyshev type inequalities, generalized proportional fractional operators, exponential function.

#### Abstract:

Inequality theory serves several new connections between mathematical analysis together with applied mathematics, statistics, engineering sciences and numerical analysis. Classical and analytical inequalities are the tools that build these connections and many researchers have made efforts to achieve strong and general inequalities. Fractional calculus have played a key role to provide novel integral inequalities by using fractional operators in recent years. In this paper, we have proved some new integral inequalities via generalized proportional fractional integral operators for integrable functions. We have obtained some more general estimations by using the expansion of exponential function.

- [1] Podlubny, I., 1999, Fractional Differential Equations, Mathematics in Science and Engineering, 198, Academic Press, New York, London, Tokyo and Toronto.
- [2] Kilbas, A. A., Srivastava, H. M. and Trujillo, J. J., 2006, *Theory and applications of fractional differential equations*, North-Holland Mathematics Studies, 204, Elsevier Sci. B.V., Amsterdam.
- [3] Jarad, F., Ugurlu, E., Abdeljawad, T. and Baleanu, D., 2017, On a new class of fractional operators, Adv. Differ. Equ., 247, doi:10.1186/s13662-017-1306-z.
- [4] Jarad F., Abdeljawad, T. and Alzabut, J., 2017, Generalized fractional derivatives generated by a class of local proportional derivatives, Eur. Phys. J. Spec. Top. 226, 34573471, https://doi.org/10.1140/epjst/e2018-00021-7.
- [5] Mitrinović, D. S., Pečarić, J. E. and Fink, A. M. 1993, Classical and new inequalities in analysis. Mathematics and its Applications, *Kluwer Academic Publishers Group, Dordrecht.*

## Equivalence Between 2-crossed Modules of R-algebroids and Simplicial R-algebroids With Moore Complex of Lenght Two

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key-words: 2-Crossed modules of R-algebroids, simplicial R-algebroids, equivalence of categories.

#### Abstract:

Group crossed modules were firstly introduced by Whitehead in [1, 2]. Crossed modules of groups are equivalent to simplicial groups with Moore complex of length one [3] and analogously for crossed modules of groupoids [4]. The notion of a 2-crossed module of groups was addressed by Conduche proved in [3] that the category of 2-crossed modules of groups is equivalent to the category of simplicial groups with Moore complex of length two.

Crossed modules and 2-crossed modules of algebras [5, 6, 7, 8] are defined in the same way as in the group case, essentially switching actions by automorphisms to actions by multipliers. As in the group case, simplicial algebras and 2-crossed modules of algebras are closely related [3, 4, 9]. If a simplicial algebra A has Moore complex of length two then a 2-crossed module can be obtained from it. This gives [8, 10, 11] an equivalence of categories from the category of simplicial algebras with Moore complex of length two and the category of 2-crossed modules of algebras.

As a more general notion, R-algebroids, where R is a commutative ring, were especially studied by Mitchell in [12, 13, 14] and by Amgott in [15]. Mitchell gave a categorical definition of R-algebroids (cf. Definition 1). Later on, as a generalisation of crossed modules of associative R-algebras, Mosa introduced crossed modules of R-algebroids and proved their equivalence to special double R-algebroids with connections in his thesis [16].

In this work, we introduce the 2-crossed module of R-algebroids, as a generalisation of 2-crossed module of commutative algebras. Then we obtain that an equivalence of categories from the category of simplicial R- algebroids with Moore complex of length two and the category of 2-crossed modules of R-algebroids.

- [1] Whitehead J. H. C. (1941), On adding relations to homotopy groups, Annals of Mathematics, 42(2), 409-428
- [2] Whitehead J. H. C. (1946), Note on a previous paper entitled "On adding relations to homotopy groups", Ann. of Math., 47(4), 806-810
- [3] Conduche D. (1984), Modules croises generalises de longueur 2, Journal of Pure and Applied Algebra, 34, 155-178
- [4] Mutlu A., Porter T., (1998), Freeness conditions for 2-crossed modules and complexes, *Theory Applications Categories*, 4, 174-194
- [5] Arvasi Z., Porter T. (1996), Simplicial and crossed resolutions of commutative algebras, Journal of Algebra, 181(2), 426-448
- [6] Arvasi Z., Porter T. (1998), Freeness conditions for 2-crossed modules of commutative algebras, Application Category Structure, 6(4), 455-471
- [7] Doncel J. L., Grandjean A.R., Vale M.J., (1992), On the homology of commutative algebras, *Journal of Pure and Applied Algebra*, 79(2), 131-157
- [8] Porter T., (1986), Homology of commutative algebras and an invariant of simis and vasconcelos, *Journal of Algebra, 99(2),* 458-465
- [9] Mutlu A., Porter T., (1998), Applications of peiffer pairings in the moore complex of a simplicial group, *Theory Applications Categories*, *4*, 148-173
- [10] Arvasi Z., (1997), Crossed squares and 2-crossed modules of commutative algebras, *Theory Applications Categories, 3*, 160-181
- [11] Grandjean A.R., Vale M.J., (1986): 2-Modulos Cruzados En La Cohomologia De Andre-Quillen, Madrid: Real Academia de Ciencias Exactas, Fisicas y Naturales de Madrid.
- [12] Mitchell B. (1972), Rings with several object, Advances in Mathematics, 8(1), 1-161
- [13] Mitchell B., (1978), Some applications of module theory to functor categories, Bull. Amer. Math. Soc., 84, 867-885
- [14] Mitchell B. (1985), Separable algebroids, Mem. Amer. Math. Soc., 57, no. 333, 96
- [15] Amgott S.M., (1986), Separable algebroids, Journal of Pure and Applied Algebra, 40, 1-14
- [16] Mosa G.H., (1986): Higher dimensional algebroids and crossed complexes, Ph. D. Thesis, University of Wales, Bangor

## Evolution of the Electric Field via Elliptical Frame: Application with Darboux Vector and Helical Trajectories

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key-words: Polarized light wave, geometric phase, electromagnetic curves, electromagnetic force.

#### Abstract:

In this study, magnetic and EM trajectories were researched, helical trajectories and elliptical Darboux vector were defined and their relationship with geometric phase (that is known as Berry's phase) was investigated through elliptical frame. In the first section, the publications, studies and developments related to the subject are given. In the second section, the geometric and physical theoretical preliminary information used in this study are given. In the third section, magnetic and E-M trajectories are calculated if the elliptical frame's vector field makes a right angle with E and these trajectories are expressed as theorems. In addition, the relationship between trajectories and special curves, which are very important in geometry, is given as a theorem. Likewise, helical trajectories have been investigated. In the fourth section, elliptic Darboux vector was defined, a new frame containing this vector was obtained, and thus elliptic magnetic and E-M trajectories were investigated. The fifth section is devoted to the results obtained in this study, and in the last section, the results obtained using the Maple program are visualized

- [1] Barros M., (1997), Magnetic helices and a theorem of Lancret, Proc. Amer. Math. Soc., 125(5), 1503-1509
- [2] Bozkurt Z., Gök İ., Yaylı Y. and Ekmekci F. N., (2014), A new approach for magnetic curves in 3D Riemannian manifolds, J. Math. Phys., 55(2014) 053501
- [3] Keskin Ö., Yayli Y., (2017), Normal Fermi-Walker Derivative, Math. Sci. and Appl. E-Notes, 5(1), 1-8
- [4] Özdenir M., (2016), An Alternative Approach to Elliptical Motion, Adv. Clifford Algebras, 26, 279-304
- [5] Özdenir Z., (2020), A New Calculus for the Treatment of Rytov's Law in the Optical Fiber, Optik International Journal for Light and Electron Optics., 216, 164892

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## Narayana Numbers as Degrees of Graphs

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key-words: Narayana numbers, degrees of vertices, Narayana graphs.

### Abstract:

To determine the conditions for a set of positive integers to be the set of vertex degrees of a graph is the main problem of our study. In this paper, we define Narayana graphs whose degree sequence consist of consecutive Narayana numbers. We determine all possible sets of vertex degrees of any arbitrary length which consist of Narayana numbers.

- [1] Aldous J. M., Wilson R. J., (2004), Graphs and Applications, An Introductory Approach, Springer, London
- [2] Delen S., Cangul I.N., (2019), Extremal Problems on Components and Loops in Graphs, Acta Mathematica Sinica, English Series, 35(2), 161-171
- [3] Delen S., Cangul I.N., (2018), A New Graph Invariant, Turkish Journal of Analysis and Number Theory, 6(1), 30-33
- [4] Demirci M., Ozbek A., Akbayrak O., Cangul I. N., (2021), Lucas Graphs, *Journal of Applied Mathematics and Computing*, 65(1-2), 93-106
- [5] Demirci M., Cangul I.N.,(2020): Tribonacci Graphs, The ITM Web of Conferences, 34, 01002, Third ICAMNM 2020
- [6] Macmahon P. A., (1915-1916): Combinatorial Analysis, Cambridge University Press
- [7] Yurttas Gunes A., Delen S., Demirci M., Cevik A.S., Cangul I.N., (2020), Fibonacci Graphs, Symmetry-Basel, 12, 1383

# **Squeezing Function and a Characterization of Polydisc**

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key-words: Squeezing function, polydisc, invariant metrics.

### Abstract:

Classical squeezing function measures how much a domain in  $\mathbb{C}^{\ltimes}$  looks like the unit ball. Recently, a modification of squeezing function was defined using polydisc instead of unit ball in  $\mathbb{C}^{\ltimes}$ . In this talk, we will first mention some well-known properties of classical and modified squeezing function. Then we will give our recent result on the characterization of polydisc using modified squeezing function.

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# An Optimal Control Strategy to Decrease the Harmful Effects of Online Game Addiction

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key-words: Mathematical model, online game addiction, optimal control.

#### Abstract:

This study focuses on determining an optimal control strategy for an online addiction model. It is a problem in which the individual cannot control himself and is excessively dependent on games on the internet. By the suggested control strategy, we aim to reduce the number of addicted individuals. The aim is to overcome addicted individuals. To support numerical results, various simulations are held by Matlab software. Graphics show the effectiveness of the proposed control strategy.

- Liu, X., Li, J., Zhao, Y., Sun, Y., and Zhang, H., (2023), A Kind of Online Game Addictive Treatment Model About Young Person, *PriMera Scientific Engineering*, vol. 2(4), pp. 36-44
- [2] Naidu, D. S., (2002): Optimal control systems, CRC press
- [3] Kirk, D. E., (2004): Optimal control theory: an introduction, Courier Corporation

## **Eigenvalues of an Impulsive Schrödinger Equation**

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key-words: Schrödinger, impulsive, eiegenvalues.

### Abstract:

In this paper, the properties of eigenvalues of an impulsive Schrödinger equation has been studied. To examine these properties, the Jost function of the problem has been analyzed. Some conditions which guarantee that the impulsive Schrödinger equation has a finite number of eigenvalues with finite multiplicities have been obtained.

## References

[1] Bainov DD., Simeovov PS., (1995), Impulsive Differential Equations: Asymptotic Properties of the Solutions, World Scientific.

- [2] Mostafazadeh A., (2011), Spectral singularities of a general point interaction, J. Phys. A: Math. Theor. 44 375302.
- [3] Ugurlu E., Bairamov E., (2013), Dissipative operators with impulsive conditions, J. Math. Chem. ; 51: 1670-1680.
- [4] Aygar Y., (2016), Investigation of spectral analysis of matrix quantum difference equations with spectral singularities., *Hacet. J. Math. Stat.*; 45: 999-1005.

## Approximation Theorems of Korovkin Type for Sequences of Monotone and Sublinear Operators via Power Series Method

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key-words: Power series method, monotone and sublinear operators, nonlinear Choquet integral.

### Abstract:

In this presentation, using power series method almost everywhere, in measure, we study Korovkin type approximation theorems for sequences of monotone and sublinear operators. We also present examples that satisfy our theorems. Finally, we calculate the rate of convergence.

- [1] Taş, E., Atlıhan, Ö.G., (2019), Korovkin type approximation theorems via power series method, *Sâo Paulo J. Math. Sci., vol.* 13, pp. 696-707
- [2] Gal, S.G., Iancu, I.T., (2023), Korovkin-Type Theorems for Statistically Convergent Sequences of Monotone and Sublinear Operators, *Bull. Malays. Math. Sci. Soc., vol. 46, pp. 1-15*
- [3] Korovkin, P.P., (1953), On convergence of linear positive operators in the space of continuous functions. (Russian), *Doklady* Akad. Nauk. SSSR. (NS), vol. 90, pp. 961-964
- [4] Anastassiou, G.A., (2018), Approximations by sublinear operators, Acta Math. Univ. Comenian., vol. 87(2), pp. 237-250
- [5] Gal, S.G., Niculescu, C.P., (2023), Korovkin type theorems for weakly nonlinar and monotone operators, *Mediterr. J. Math.*, vol. 20, pp. 1-20

## **On Invariant Subspace of the Shift Operator on** $\mathcal{D}(\mu)$ **Space**

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key-words: Weighted Dirichlet spaces, invariant subspaces.

### Abstract:

The process of reconstructing a subspace not only by the root vectors that are contained in it but also by the limit process of a sequence of subspaces, called approximate spectral synthesis, was suggested by Nikolskii [2, 3]. To be more precise, let X be a Banach space of analytic functions on the unit disk  $\mathbb{D}$ . Suppose X is invariant with respect to the shift operator S. A subspace M of X, which is invariant with respect to the shift operator S, is said to admit strong approximate spectral cosynthesis if there exists a sequence  $M_n$  of invariant subspaces such that  $\dim(X/M_n) < \infty$ ,  $M = \underline{\lim}M_n$ , and  $M^{\perp} = \underline{\lim}M_n^{\perp}$ , where  $\underline{\lim}M_n = \{x \in X : \exists x_n \in M_n \text{ with } x_n \to x\}$  (see Definition 4 of [3]). In this talk, we discuss the result that proves that shift invariant subspaces of  $\mathcal{D}(\mu)$  has this property whenever the associated measure  $\mu$  has finite support.

- [1] D. Guillot, (2012), Fine boundary behavior and invariant subspaces of harmonically weighted Dirichlet spaces, Complex Anal. Oper. Theory 6(6), 1211–1230.
- [2] N. K. Nikolski, (1984), Two problems on spectral synthesis, Journal of Soviet Mathematics 26(5), 2185–2186.
- [3] S. M. Shimorin, (2000), Approximate spectral synthesis in the Bergman space, Duke Math. J. 101(1), 1–39.

## **Barr Exactness of Quadratic Modules over Nil(2)-Modules**

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key-words: Quadratic module, exactness, pullback.

### Abstract:

A crossed module given by Whitehead in [4] is an algebraic representation of a 2-type with the lowest degree of Peiffer nilpotency. The optimum algebraic system for representing a 2-type is crossed modules. Conduché used simplicial properties to define 2-crossed modules. Crossed squares [3], 2-crossed modules [1], quadratic modules are all algebraic equivalent of a 3-type. The categorical relations of quadratic modules can be summarized with the following diagram



In this work, we investigate the Barr exactness [2] of the quadratic modules over the same nil(2)-module base. In this manner, we give the product of the quadratic modules and obtain that this category is finitely complete. Then we show that the the category of quadratic modules exact by means of Barr.

- [1] Conduché D., (1984), Modules croisés généralisés de longueur 2, J. Pure and Applied Algebra, 34, 155-178
- [2] Barr M., Grillet P., and Van Osdol D., (1971): Exact categories and categories of sheaves, Publisher Springer Verlag
- [3] Guin-Waléry D., and Loday J.L., (1981), Obstructioná l'excision en K-théorie algébrique, in: Algebraic K-Theory, Lecture Notes in Math, 854, 179-216
- [4] Whitehead J.H.C., (1949), Combinatorial homotopy II, Bull. Amer. Math. Soc., 55, 453-496

## A New Class of Exact Penalty Function Method for Inequality Constrained Optimization Problems

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key-words: Non-linear programming, exact penalty function, smoothing technique.

#### Abstract:

This study introduce a new family of smoothing techniques for a new class of exact penalty functions, which transforms the constrained optimization problems into unconstrained optimization problems. It is proved that each optimal solution of the exact penalty function corresponds to an optimal solution of the original problem. Error analysis is performed to prove that the optimal solution of the smoothed exact penalty problem is an optimal solution to the constrained optimization problem. Based on the proposed smoothing technique, an algorithm which generates an optimal solution to the constrained optimization problem is presented. The convergence of the proposed algorithm is proved in terms of theoretical and numerical aspects. Some numerical examples are given to demonstrate the efficiency of the presented smoothing exact penalty method.

- Meng, Z., Dang, C., Jiang, M. and Shen, R., 2011, A smoothing objective penalty function algorithm for inequality constrained optimization problems, *Numerical Functional Analysis and Optimization, vol. 32(7), pp. 806-820.*
- [2] Xu, X., Dang, C., Chan, F. T. and Wang, Y., 2019, On smoothing  $l_1$  exact penalty function for constrained optimization problems, *Numerical Functional Analysis and Optimization, vol.* 40(1), pp. 1-18.
- [3] Qiu, J., Yu, J. and Lian, S., 2021, Smoothing Approximation to the New Exact Penalty Function with Two Parameters, Asia-Pacific Journal of Operational Research, vol. 38(5), 2140010.
- [4] Yilmaz, N., Ogut, H., 2023, An exact penalty function approach for inequality constrained optimization problems based on a new smoothing technique, *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, In Press.*

## **Designing a PSO-Trained ANN Model for Estimation of Tourism Incomes**

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key-words: Artificial Neural Network, estimation of incomes, Particle Swarm optimization.

#### Abstract:

Artificial neural network (ANN) is one of the artificial intelligence techniques widely used in estimation studies recently. To obtain effective results with ANN, determining the variables, such as the input structure, the number of hidden layers, and the number of units in the hidden layer, which form the architectural structure of the ANN and the learning algorithms to be used in the training of the ANN is significant. A common conclusion has yet to be reached in determining these factors in the studies carried out. Therefore, reviewing these factors for each problem is necessary to obtain good predictive results. In this study, the training of ANN was carried out using the Particle Swarm Optimization (PSO) algorithm, which is one of the well-known metaheuristic methods, and a PSO-trained ANN model was designed to estimate tourism incomes by comparing the estimation performances of various architectural structures. In addition, the PSO-trained ANN model's results were compared with those of the manually designed ANN model using Backpropagation and Levenberg-Marquardt (BP-LM) learning algorithms. It has been observed that the PSO-trained ANN is generally more successful than the BP-LM-trained ANN algorithm according to MSE, RMSE, and MAE performance criteria for estimating tourism incomes.

- [1] Zhang, J. R., Zhang, J., Lok, T. M., and Lyu, M. R. (2007), A hybrid particle swarm optimization–back-propagation algorithm for feedforward neural network training, *Applied mathematics and computation, vol. 185(2), pp.1026-1037*
- [2] Fadlallah, S. O., Anderson, T. N., and Nates, R. J. (2021), Artificial neural network-particle swarm optimization (ANN-PSO) approach for behaviour prediction and structural optimization of lightweight sandwich composite heliostats, *Arabian Journal for Science and Engineering*, vol. 46, pp.12721-12742.